DESIGNING OPTIMAL ROAD PRICING SCHEMES FOR DIFFERENT POLICY OBJECTIVES WITH HETEROGENEOUS TRAVELERS

Dusica Joksimovic, Delft University of Technology, Faculty of Civil Engineering and Geosciences, Transport & Planning Section, d.joksimovic@citg.tudelft.nl

Michiel C.J. Bliemer, Delft University of Technology, Faculty of Civil Engineering and Geosciences, Transport & Planning Section, m.bliemer@citg.tudelft.nl

Bijdrage aan het Colloquium Vervoersplanologisch Speurwerk 2005,

24 en 25 november 2005, Antwerpen

Contents

Samenvatting

Ontwerpen van Optimale Beprijzingsstrategieën voor Verschillende Beleidsdoelstellingen met Heterogene Reizigers

In vervoersnetwerken maken reizigers route- en vertrektijdstipkeuzes die in het algemeen niet optimaal zijn voor bepaalde beleidsdoelstellingen van de wegbeheerder. Door verschillende beprijzingsstrategieën te introduceren kunnen deze doelstellingen eventueel worden geoptimaliseerd. In dit artikel worden verschillende prijsscenario's geanalyseerd (uniform, quasi-uniform, variabel) gebruik makend van een dynamisch verkeersmodel. We analyseren maximalisering van de totale tolopbrengsten en minimalisering van de totale reistijd op het netwerk als beleidsdoelstellingen. Dit ontwerp probleem is geformuleerd als een bi-level optimalisatie probleem waarbij op het hoogste niveau de beleidsdoelstellingen beschrijven van de wegbeheerder met gekozen beprijzingsstrategie, terwijl op het laagste niveau het dynamisch verkeerstoedelings-model wordt beschreven. Heterogene reizigers worden verondersteld met een hoge en lage reistijdwaardering. Het ontwerp probleem wordt opgelost door middel van een eenvoudig 'grid search' algoritme. In case studies op een eenvoudig hypothetisch netwerk wordt aangetoond dat beleidsdoelstellingen inderdaad kunnen worden geoptimaliseerd door middel van beprijzing, en dan verschillende beleidsdoelstellingen leiden tot verschillende optimale prijsscenario's en hoogtes van de tolheffingen.

Summary

Designing Optimal Road Pricing Schemes for Different Policy Objectives with Heterogeneous Travelers

In transport networks, travelers individually make route and departure time choices that may not be optimal for a specific policy objective of the road authority. By introducing different tolling strategies, this policy objective may be optimized. In the paper, the effects of different tolling schemes (uniform, quasi-uniform and variable) will be analyzed using a dynamic traffic model. We analyze maximizing total toll revenues and minimizing total travel time on the network as policy objectives. The optimal toll network design problem is formulated as a bi-level optimization problem in which the upper level describes policy objectives of the road authority with chosen toll strategy while the lower level describes the dynamic traffic assignment model. Heterogeneous travelers are assumed having a low and high value of time. The optimal toll network design problem is solved using a simple grid search algorithm. In case studies on a simple hypothetical network it is shown that policy objectives can indeed be optimized by imposing tolls, and that different policy objectives lead to different optimal tolling schemes and toll levels.

1 Introduction

1.1 Background

Road pricing presents one of the market-based policy instruments having influence on travel behavior of users of a transportation network. Road pricing is a type of responsive pricing that can change travel patterns by influencing users' travel choices at various levels (e.g. departure time choice, route choice). Many researches have been working on road pricing problems (see e.g. Verhoef, 2002; May and Milne, 2000), though almost all of the modeling studies consider static models. Dynamic models describe the problem more accurate and are required for studies that look at time-varying road pricing. However, formulating and solving dynamic models with time-varying pricing is much more complex compared to static models.

In this paper we are dealing with the time-varying optimal toll design problem for planning purposes. Uniform and variable (time-varying) tolls during the peak are considered, in which travelers' responses (route choice and departure time choice) to these tolls are taken into account. For simplicity, the total number of travelers is assumed constant. In this paper we consider so-called second best pricing, which means that only a subset of road segments is tolled, in contrast to first-best pricing in which all links are tolled according to their marginal costs (see e.g. Verhoef, 2000). The planning application means that if the road authority plans to adopt road pricing for its purposes, then the analyst can assess whether such a toll pattern can help achieving policy objectives. In addition, the question of what the optimal time-varying toll pattern should like (distribution of tolls over time and space) and what the road authority will achieve applying different policy objectives should be answered. The problem of finding the optimal tolls can be formulated as a network design problem.

The focus of this paper is to describe the framework of the optimal toll network design problem and to mathematically formulate the problem. Furthermore, it will be illustrated that different objectives of the road authority and different tolling schemes can lead to different optimal toll levels. The main contributions of this paper are the following. First, a dynamic instead of static model with road pricing is proposed, which includes not only route choice but also departure time choice. Secondly, heterogeneous travelers with high and low value of time are considered. Thirdly, different objectives of the road authority are explored, namely maximizing revenues and minimizing total network travel time. Finally, different tolling schemes and their impact on the objective of the road authority are analyzed.

1.2 Literature study

The problem of road pricing has been studied in the literature from different modeling perspectives and under various assumptions. The (economical) theory of road pricing dates back to Pigou (1920) and first-best congestion tolls are derived in static deterministic models (Beckmann et al., 1956; Dafermos, 1973; Yang and Huang, 1998) and static stochastic models (Yang , 1999).

Dynamic pricing models in which network conditions and link tolls are time-varying, have been addressed in Wie and Tobin (1998), comparing the effectiveness of various pricing policies (time-varying, uniform and step-tolls). A limitation of these models is that they are restricted to a bottleneck or a single destination network. Mahmassani and Herman (1984) and Ben-Akiva et al. (1986) developed dynamic marginal (first-best) cost pricing models for general transportation networks. As indicated by the authors, the application of their model is limited to destination-specific (rather than route or link-based) tolling strategies, which might not be easy to implement in practice. Moreover, since tolls are based on marginal cost pricing, it is implicitly assumed that all links can be priced, which is practically infeasible.

Second-best pricing models typically only toll a subset of links. In Viti et al. (2003) a dynamic congestion-pricing model is formulated as a bi-level programming problem, and the prices are allowed to affect the (sequentially) modeled route and departure time choice of travelers. Abou-Zeid (2003) developed some models for pricing in dynamic traffic networks. In Joksimovic et al. (2005) the time-varying pricing problem including route and departure choice is solved using a simple algorithm for the road authority’s objective of minimizing total travel time on the network. In this paper these models are extended to include heterogeneous travelers, different and more general tolling schemes, and different objectives of the road authority.

1.3 Structure of the paper

In Section 2 the optimal toll network design framework and all components are discussed and mathematically formulated. Then, in Section 3 a simple solution algorithm is proposed. Section 4 illustrates and discusses on a small hypothesized network how the model works under different tolling schemes and different objectives of the road authority. Finally, conclusions are drawn in Section 5.

2 The model

Let $G=(N,A)$ denote a given transport network G with nodes N and directed links A. Furthermore, let D_m^{rs} describe the given travel demand (total number of travelers) for each origin-destination (OD) pair (r,s) and for each user class $m \in M$. Between each OD pair there exist routes $p \in P^{rs}$, where P^{rs} denotes the set of all feasible routes. All user classes are assumed to use the same infrastructure. The total time horizon considered is denoted by set T, and each time interval is denoted by $t \in T$. The travel demand interval is defined by set $K \subset T$, where each $k \in K$ is a feasible departure time interval.

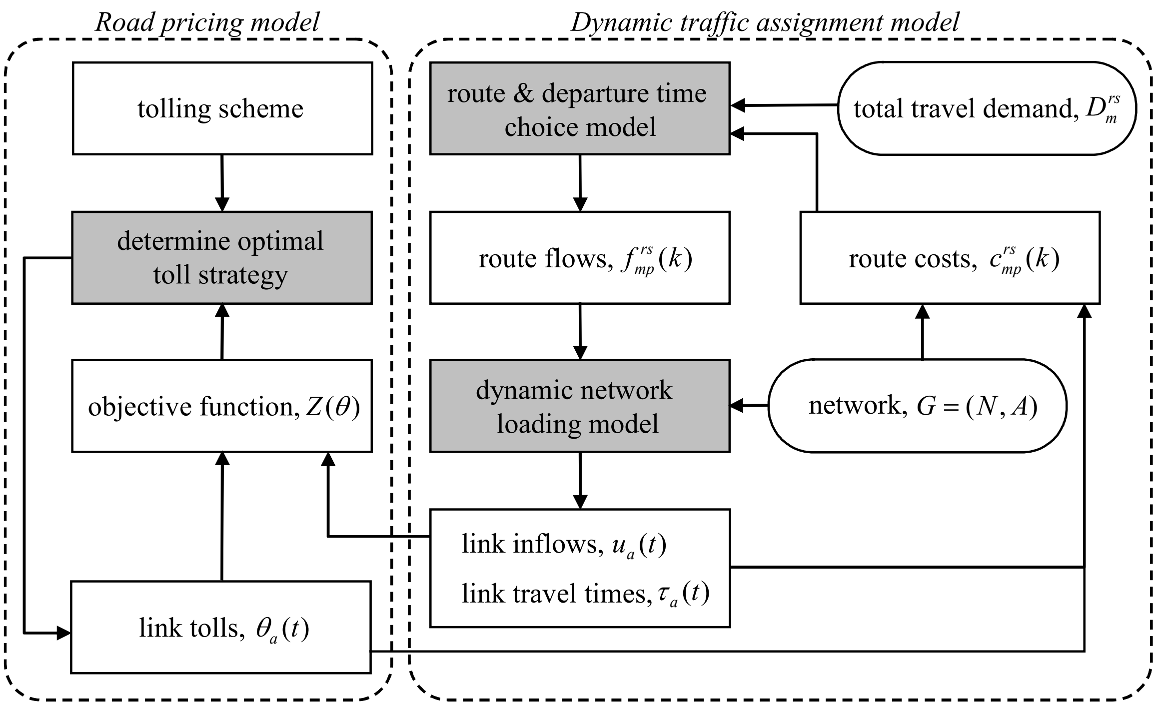

Figure 1: Optimal toll design model framework

The aim of the *optimal toll design* model is to determine optimal tolls for a certain toll scheme and a certain objective of the road authority, given the departure time and route choice decisions of the travelers. The model framework is depicted in Figure 1. It consists of two main parts, namely a road pricing part and a dynamic traffic assignment part. This framework essentially describes a bi-level problem. The upper level is the road pricing part, while the lower level is the dynamic traffic assignment for given toll levels. Both parts will be explained in more detail in the following subsections.

2.1 Road pricing model

In the road pricing model the road authority aims to introduce the best tolling strategy depending on their goals. The road authority may have different goals, leading to different objective functions in the model. Depending on the goal, the road authority has to select the best tolling scheme with the best toll levels. First, some potential objective functions are discussed. Secondly, a few tolling schemes used in this paper will be introduced.

Objectives

For our experiments, two different objectives are chose, namely (1) maximization of revenues, and (2) minimization of total travel time on the network.

Objective function $Z_{\text{revenue}}(\theta)$ describes the total revenues on the network to be maximized, depending on the link tolling levels $\theta \equiv \left[\theta_a(t)\right]$,

$$\max_{\theta\in\Theta} Z_{\text{revenue}}(\theta) = \sum_a \sum_t u_a(t)\theta_a(t), \tag{1}$$

where $\theta_a(t)$ is the toll level on link a at time t (according to the different tolling schemes described below), and $u_a(t)$ is the number of vehicles flowing into link a at time t, taking into account that the travelers are confronted with the toll levels. The set Θ denotes the set of feasible toll levels, which may include upper and lower bounds, etc. In this paper we consider a so-called second-best tolling strategy. This means that some links may not be tolled, e.g. the toll levels are then constrained to zero for the untolled links for all time intervals.

Objective function $Z_{\text{time}}(\theta)$ describes the total travel time on the network to be minimized, depending on the link tolling levels θ,

$$\min_{\theta \in \Theta} Z_{\text{time}}(\theta) = \sum_a \sum_t u_a(t) \tau_a(t), \tag{2}$$

where $\tau_a(t)$ is the link travel time when entering link *a* at time *t*.

Tolling schemes

Different tolling schemes can be considered by the road authority. As illustrated in Figure 2, we distinguish (1) a uniform tolling scheme (toll levels are constant over the entire study time period), (2) a quasi-uniform tolling scheme (tolls levels are constant over a specified time period and zero otherwise), and (3) a variable tolling scheme (tolls levels are time-varying).

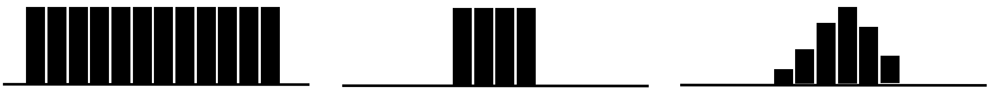

Figure 2: Different tolling schemes

The three different tolling schemes can be formulated as follows:

Uniform: $$\theta_a(t) = \overline{\theta}_a, \quad \forall t. \tag{3}$$

Quasi-uniform: $$\theta_a(t) = \begin{cases} \overline{\theta}_a, & \text{if } t \in \overline{T} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Variable: $$\theta_a(t) = \phi_a(t)\overline{\theta}_a, \quad \forall t. \tag{5}$$

The value $\overline{\theta}_a$ denotes a given toll value for each link *a*. In case of uniform tolls, the toll levels for all time periods are set to this toll value. In case of quasi-uniform tolls, only the tolls in the time periods $t \in \overline{T} \subset T$ will be set to the toll value $\overline{\theta}_a$, and will be zero outside that time period. For variable tolls we assume there is a given predefined function $\phi_a(t)$ over time for each link. In other words, the proportions of the time-varying tolls are fixed (hence, the shape of the toll levels over time is given), such that there is still only a single toll value $\overline{\theta}_a$ to be determined.

For each evaluation of the objective functions $Z_{\text{revenue}}(\theta)$ and $Z_{\text{time}}(\theta)$ a dynamic traffic assignment problem has to be solved (see Figure 1).

2.2 Dynamic traffic assignment model

The dynamic traffic assignment (DTA) model consists of two components: (1) a simultaneous route choice and departure time choice component, and (2) a dynamic network loading component. In the route and departure time choice component, travelers are modeled as utility maximizers in which they choose the route and departure time that minimizes a certain generalized costs, yielding dynamic route flows. The dynamic loading component then dynamically propagates these route flows over the network, yielding (new) experienced travel times and toll costs. The interaction between the two components is depicted in Figure 1. In this paper the problem is considered in discrete time, hence defined in terms of time intervals instead of time instants. Both components will be explained in more detail below.

Simultaneous route choice and departure time choice component

Let the class m experienced generalized travel cost for each route p from origin r to destination s and departure time interval k (denoted by $c_{mp}^{rs}(k)$) be given by a linear combination consisting of the route travel time $\tau_p^{rs}(k)$, penalties for scheduling delays, and route toll costs $\theta_p^{rs}(k)$:

$$c_{mp}^{rs}(k) = \alpha_m \tau_p^{rs}(k) + \beta \left| k - \varsigma^{rs} \right| + \gamma \left| k + \tau_p^{rs}(k) - \xi^{rs} \right| + \theta_p^{rs}(k), \tag{6}$$

where $k - \varsigma^{rs}$ denotes the deviation from the actual departure time k and the preferred departure time ς^{rs}, and where $k + \tau_p^{rs}(k) - \xi^{rs}$ denotes the deviation from the actual arrival time $k + \tau_p^{rs}(k)$ and the preferred arrival time ξ^{rs}. The parameters α_m, β, and γ convert time to a monetary value. Parameter α_m denotes the value-of-time (VOT) of class m travelers. Later on, we will distinguish high and low VOT travelers. Note that the VOT is the only class-specific parameter. The travel times and tolls are assumed to be the same for all traveler types. Hence, we are not assuming different vehicle types, but focus on travelers making e.g. business trips (high VOT) versus leisure trips (low VOT).

The route travel times and the route toll costs are determined from the corresponding link travel times and toll costs along the route. Let $\delta_{ap}^{rs}(k,t)$ be a dynamic route-link incidence indicator, which is one if link *a* is reached on route *p* from *r* to *s* at time *t* when departing at time *k*, and zero otherwise. This indicator can be computed when the link travel times $\tau_a(t)$ are known. Then the route travel time can be computed from consecutive link travel times,

$$\tau_p^{rs}(k) = \sum_{a \in p} \tau_a(t) \delta_{ap}^{rs}(k,t). \tag{7}$$

Similarly, the route toll costs can be computed from consecutive link toll costs $\theta_a(t)$,

$$\theta_p^{rs}(k) = \sum_{a \in p} \theta_a(t) \delta_{ap}^{rs}(k,t). \tag{8}$$

Based on the experienced generalized travel costs $c_{mp}^{rs}(k)$, each traveler is assumed to choose the route and departure time that he or she perceives has the least travel costs, yielding a stochastic user-equilibrium assignment. Assuming that the random components on the generalizes travel costs are independently (which may not hold if routes overlap) and identically extreme value type I distributed, then the joint probability of choosing route *p* and departure time *k* is given by the following multinomial logit (MNL) model:

$$\psi_{mp}^{rs}(k) = \frac{\exp\left(-\mu c_{mp}^{rs}(k)\right)}{\sum_{p' \in P^{rs}} \sum_{k'} \exp\left(-\mu c_{mp'}^{rs}(k')\right)}, \quad \forall (r,s), p \in P^{rs}, m. \tag{9}$$

Given the travel demand D_m^{rs}, the dynamic class-specific route flows can be determined by

$$f_{mp}^{rs}(k) = \psi_{mp}^{rs}(k) D_m^{rs}, \quad \forall (r,s), p \in P^{rs},\ m. \tag{10}$$

The DTA model is basically a fixed-point problem. Generalized route travel costs yield route flows, while route flow affect the travel times and therefore the generalized route travel costs. The relationship between the route flows and the travel times is given by the dynamic network loading component.

Dynamic network loading component

The dynamic network loading (DNL) component 'simulates' the route flows on the network, yielding link flows, link volumes, and link travel times. The DNL model used in this paper is a very simple system of equations adapted from Chabini (2000) and Bliemer and Bovy (2003) in which the flow propagation equation is simplified by assuming that there are no subintervals within one time interval and that the link travel time is stationary. In this case, the equations are similar to the ones proposed by Ran and Boyce (1996).

The following set of equations describe the dynamic network loading model:

$$v_{ap}^{rs}(t+\tilde{\tau}_a(t)) = u_{ap}^{rs}(t), \tag{11}$$

$$u_{ap}^{rs}(t) = \begin{cases} \sum_m f_{mp}^{rs}(t), & \text{if } a \text{ is the first link on route } p, \\ v_{a'p}^{rs}(t), & \text{if } a' \text{ is the previous link on route } p. \end{cases} \tag{12}$$

$$u_a(t) = \sum_{(r,s)} \sum_{p \in P^{rs}} u_{ap}^{rs}(t). \tag{13}$$

$$v_a(t) = \sum_{(r,s)} \sum_{p \in P^{rs}} v_{ap}^{rs}(t). \tag{14}$$

$$x_a(t) = \sum_{w \le t} u_a(w) - v_a(w). \tag{15}$$

$$\tau_a(t) = \tau_a^0 + b_a x_a(t). \tag{16}$$

The flow propagation equations in Eqn. (11), which determine describe the propagation of the inflows through the link and therefore determine the outflows, relate the inflows $u_{ap}^{rs}(t)$ and outflows $v_{ap}^{rs}(t)$ of link a at time interval t of vehicles traveling on route p from r to s, respectively. This equation simply states that traffic that enters link a at time t will exit the link when the link travel time $\tau_a(t)$ elapses. Note that since we are dealing with a discrete-time problem, the link exit time $t+\tau_a(t)$ needs to be an integer value. Therefore, $\tilde{\tau}_a(t)$ is used, which simply rounds off the travel time (expressed in time intervals) to the nearest integer.

Eqn. (12) describe the flow conservation equations. If link a is the first link on a route, the inflow rate is equal to the corresponding route flows determined by the simultaneous route and departure time choice model. Since we have assumed that all vehicles travel at the same speed, the low and high VOT users can be combined in the DNL model by summing them up. If link a is not the first link on a route, then the link inflow rate is equal to the link outflow rate of the previous link.

Eqns. (13)–(15) are definitions. The first two simply stating that the total link inflows $u_a(t)$ (or outflows $v_a(t)$) are determined by adding all link inflows (or outflows) for all routes that flow into (out of) link a at that time interval. Eqn (15) defines the number of vehicles on link a at the beginning of time interval t, $x_a(t)$, which is by definition equal to the total number of vehicles that have entered the link until time interval t, $\sum_{w\leq t} u_a(w)$, minus the total number of vehicles that have exited the link, $\sum_{w\leq t} v_a(w)$. Finally, Eqn. (16) relates for each link a the number of vehicles to the travel time on that link as an increasing function, where each link has a free-flow travel time τ_a^0, and a delay component $b_a x_a(t)$, (with b_a a nonnegative parameter).

3 Solution algorithm

Each component of the optimal toll design problem can be solved using various types of algorithms. The outline of the complete algorithm for the case of variable tolls is as follows. The algorithm starts with specifying the grid of considered decision values (toll levels) satisfying the constraints (the lower and upper bounds of the tolls). In each iteration, the algorithm solves a DTA problem (i.e. finds a dynamic stochastic multiclass user equilibrium solution) based on the current toll levels and sets new tolls that can potentially optimize the objective functions described in Eqn. (1) or Eqn. (2). Because the algorithm is a grid-search method it stops after all feasible toll values in the grid have been considered. At this stage of the research, the focus is mainly to investigate the framework of the model and the properties of the solutions for different objectives and tolling schemes, and not on the development of algorithms. More efficient algorithms will be developed in the future research. The two-stage iterative grid-search procedure for the optimal time-varying toll problem with DTA (including joint route and departure time choice) can be outlined as follows:

Outer loop: PRICING

Step 1: *Initialization*

Consider a certain tolling scheme in Eqns. (3)–(5). Specify set of grid points $\overline{\theta}^{(i)} \equiv [\overline{\theta}_a^{(i)}]$, $i = 1, \ldots, N,$ to be evaluated, satisfying the constraints in set Θ. Set $i := 1$ and set $Z^* = \pm\infty$ (depending on whether the objective should be minimized or maximized).

Step 2*: Set toll values*

Select grid point *i* for the toll levels, yielding tolls $\theta_a^{(i)}(t)$ from Eqns. (3)–(5).

Inner loop: DTA

Step 3a: *Initialization* – Set $j := 1$. Assume an empty network and free-flow network conditions, i.e. $\tau_a^{(j)}(t) = \tau_a^0$.

Step 3b: *Compute dynamic route costs* – Compute travel costs $c_{mp}^{rs,(j)}(k)$ using Eqns. (6)–(8).

Step 3c: *Compute new intermediate route flows* – Determine the new intermediate dynamic route flow pattern $\tilde{f}_{mp}^{rs,(j)}(k)$ using Eqns. (9)–(10).

Step 3d*: Flow averaging* – Use the Method of Successive Averages (MSA) to update the route flows: $f_{mp}^{rs,(j)}(k) = f_{mp}^{rs,(j)}(k) + \frac{1}{j}\left(\tilde{f}_{mp}^{rs,(j)}(k) - f_{mp}^{rs,(j)}(k)\right)$.

Step 3e: *Perform dynamic network loading* – Dynamically load $f_{mp}^{rs,(j)}(k)$ onto the network using Eqns. (11)–(16), yielding new link travel times $\tau_a^{(j+1)}(t)$.

Step 3f: *Convergence of DTA level* – If the dynamic duality gap is sufficiently small, go to Step 4; otherwise set $j := j + 1$ and return to Step 3b.

Step 4: *Compute objective function*

Compute the objective function $Z(\theta^{(i)})$ using Eqn. (1) or Eqn. (2). If $Z(\theta^{(i)})$ is better than Z^*, then set $Z^* = Z(\theta^{(i)})$ and set $\overline{\theta}^* = \overline{\theta}^{(i)}$.

Step 5: *Convergence of road pricing level*

While $i < N$, set $i := i + 1$ and return to Step 2. Otherwise, the algorithm is terminated and $\overline{\theta}^*$ is the set of optimal toll levels.

Performing this simple iterative procedure, we explore all possibilities for all toll level combinations and find the optimal value of the objective function. Regarding the convergence of this algorithm, the inner DTA loop using the widely used heuristic MSA procedure typically converges to an equilibrium solution, although convergence cannot be proven. In the outer road pricing loop the whole solution space is investigated with a certain grid accuracy (yielding a finite number of solutions that are evaluated).

4 Case studies

4.1 Network description, travel demand and input parameters

The solution procedure proposed in the previous section has been applied to a small network, see Figure 3. The network consists of just a single OD-pair connected by two non-overlapping paths where only link 2 is tolled. Since there is only one OD pair, we will ignore the OD subindices (r,s) in the variables. Two user classes with different value of time (VOT) are distinguished. The total travel demand for departure period $K=\{1,\ldots,20\}$ from node 1 to node 3 is $D=86,$ of which 50% high VOT travelers and 50% low VOT travelers. The following parameter values are used on route level: preferred departure time $\varsigma=10,$ preferred arrival time $\xi=15$, value of time for class 1 $\alpha_1=0.25,$ value of time for class 2 $\alpha_2=0.75,$ penalty for deviating from preferred departure time $\beta=0.25,$ penalty for deviating from preferred arrival time $\gamma=1,$ and scale parameter in MNL model $\mu=0.8.$ On the link level, we assume that route 1 with a free-flow travel time of 7.0 time intervals is longer than route 2 (3.0 time intervals) by setting the free-flow link travel times in Eqn. (16) to $\tau_1^0=\tau_3^0=3.5$ and $\tau_2^0=3.0.$ Furthermore, it is assumed that the first route never have congestion, hence $b_1=b_3=0,$ while congestion is possible on link 2 for which we set $b_2=0.005.$

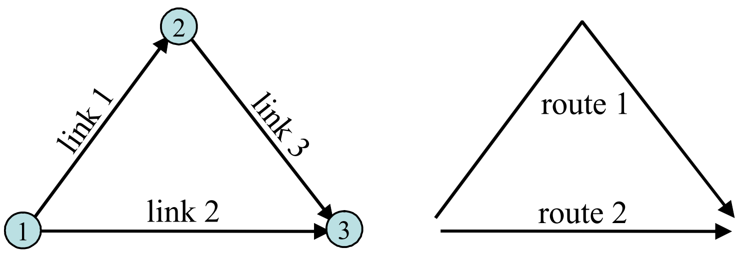

Figure 3: Network for case studies

For the case in which the tolls are zero on link 2, the route flows and costs are depicted in Figure 4. The flows are almost evenly spread between the two routes. The departure time profiles indicate that travelers that use the longer route 1 will depart earlier in order to arrive as close as possible to their preferred arrival time. Since high VOT users attach a higher weight to the travel time in their cost, more high VOT users will be using route 2 as this route will typically have a lower travel time (even with some congestion), whereas route 1 is primarily used by low VOT users who take mainly the penalty for arriving late or early at the destination into account.

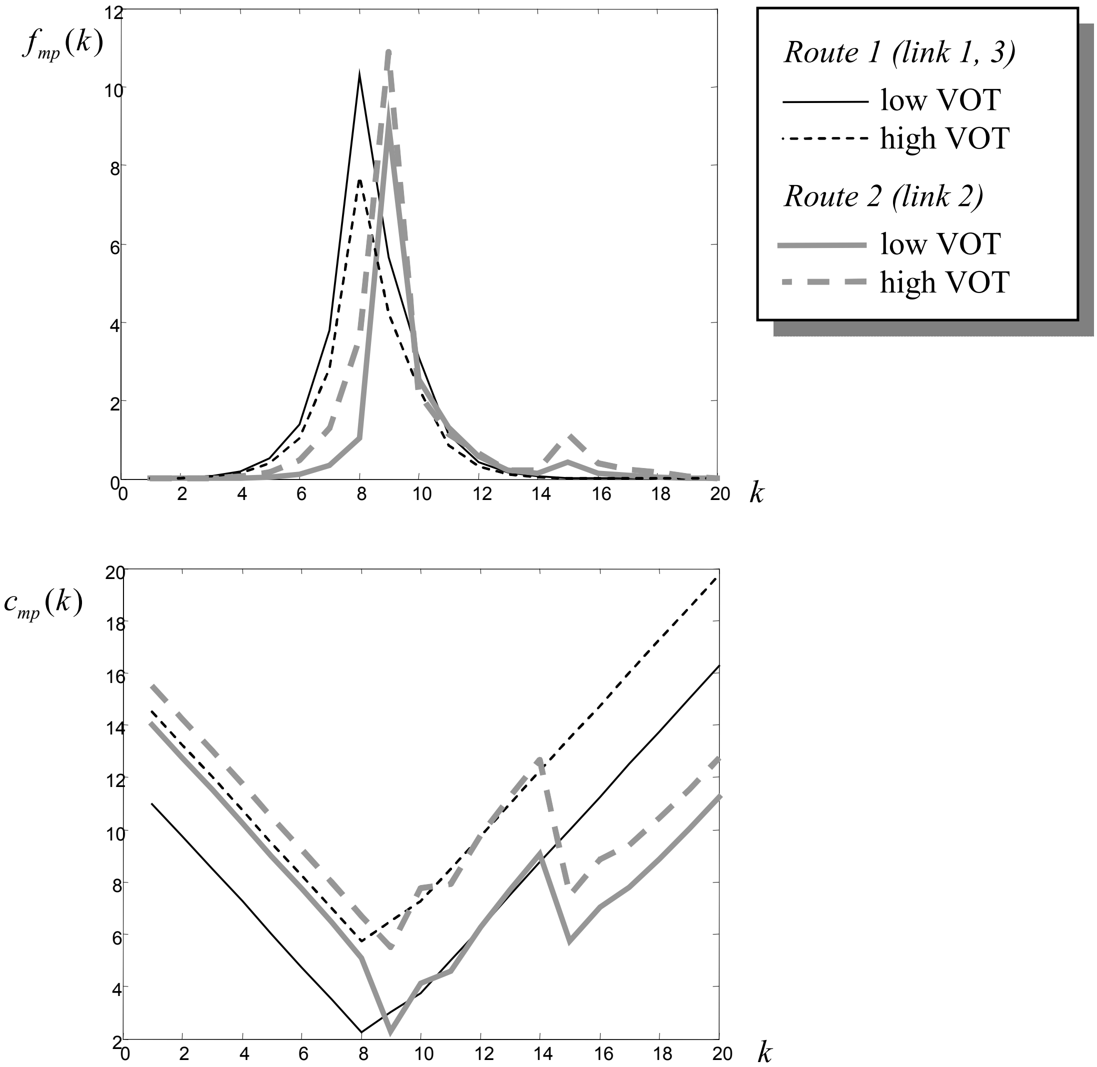

Figure 4: Route flows and costs in the case of zero tolls

In Section 2.1 three different tolling schemes have been mentioned, namely the uniform, the quasi-uniform, and the variable tolling scheme, see Eqns. (3)–(5). All three tolling regimes will be considered in the case studies below. Note that in this case study with only a single link (link 2) tolled, determining the optimal toll for each tolling regime (even for the variable tolling scheme) only requires to find a single optimum toll level, $\overline{\theta}_2^*$. The uniform tolling scheme does not have any parameters. For the quasi-uniform tolling scheme we assume that a toll will be levied in the peak period, i.e. the tolling period is $\overline{T} = \{8,9,10,11,12\}$ in Eqn. (4). In the variable tolling scheme only periods 9, 10, and 11 will be tolled with fixed proportions 0.6, 1.0, and 0.6, that is

$$\phi_2(t) = \begin{cases} 1.0, & \text{if } t = 10; \\ 0.6, & \text{if } t = 9,11; \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

4.2 Objective: Maximize total revenues

Assume that the road authority aims to maximize total revenues, as formulated in Eqn. (1), by selecting the best tolling scheme and the best toll level. The three different tolling regimes as mentioned above will be considered. For each tolling scheme and for each toll level, the dynamic traffic assignment (DTA) problem can be solved. In Figure 5 the total revenues are plotted for each tolling scheme for all $0 \leq \overline{\theta}_2 \leq 15$ (although not shown here, in all cases the DTA model converged). In case the toll level is zero, there are clearly no revenues. For very high toll levels, all travelers will choose to travel on the untolled route, resulting in zero revenues as well. As can be observed from Figure 5, uniform tolling with $\overline{\theta}_2 = 3.11$ yields the highest revenues. The variable tolling scheme is not able to provide high revenues due to the small number of tolled time periods.

The route flows and costs are also depicted in Figure 6, together with the optimal toll levels for the objective of maximizing revenues. Compared with the case of zero tolls in Figure 4, it can be seen that travelers shift towards (non-congested and untolled) route 1 and also shift their departure time (mostly later). Furthermore, it can be observed that there are many more travelers with a high VOT tolled route 2 than travelers with a low VOT. This is to be expected, as travelers with a high VOT care less about toll costs and more about a short trip time.

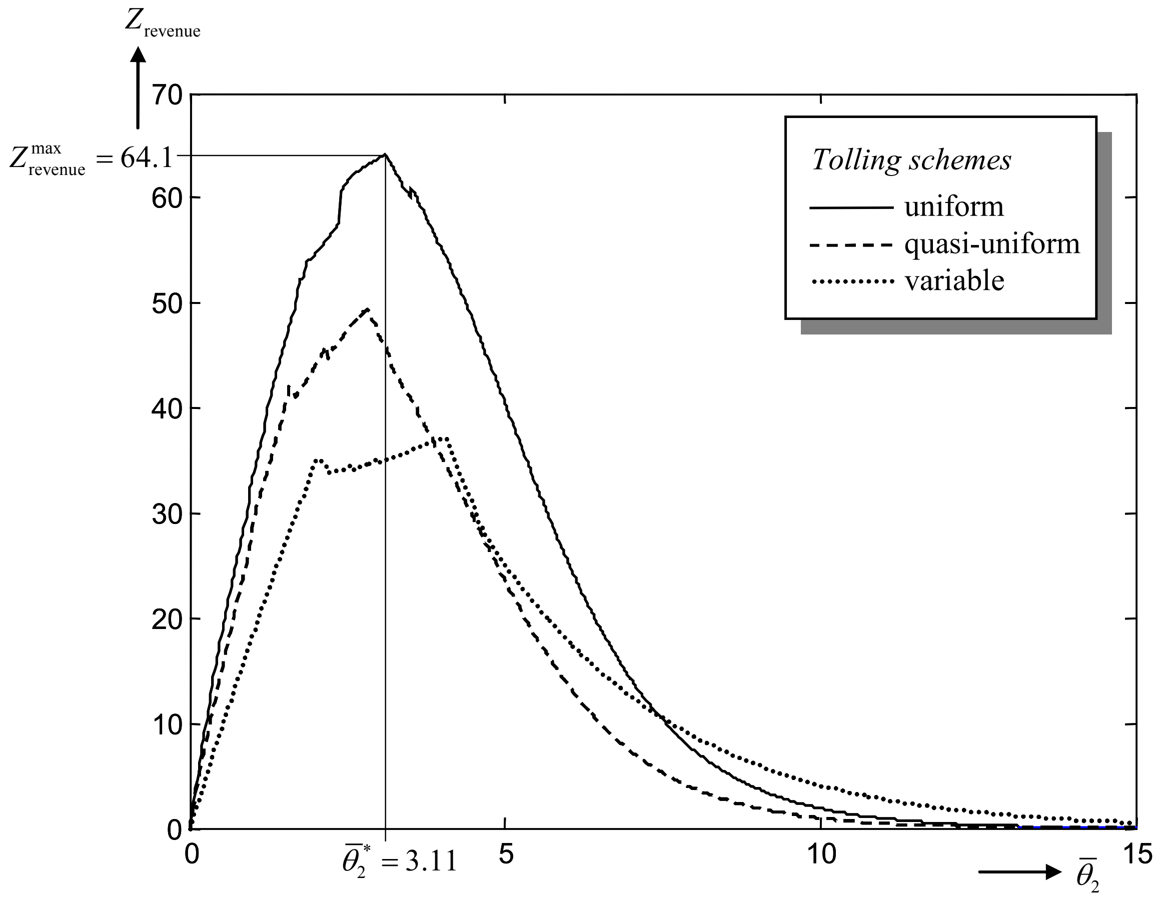

Figure 5: Total revenue for different tolling schemes and toll levels

4.3 Objective: Minimize total travel time

In this case study, the road authority aims at minimizing total travel time on the network (see Eqn (2)) by selecting the best tolling scheme and the best toll level. Figure 7 depicts the total travel times for different tolling schemes and toll levels.

As can be observed from Figure 7, it seems possible to decrease the total travel time on the network by imposing a toll on congested route 2. High toll levels will push all travelers during the tolled period away from route 2 to the longer route 1, yielding higher total travel times again. Variable tolling with $\overline{\theta}_2^* = 3.24$ (yielding $\theta_2^*(10) = 3.24$ and $\theta_2^*(9) = \theta_2^*(11) = 1.99$ according to Eqns. (5) and (17)) results in the lowest total travel time. The objective function looks somewhat irregular. However, this can be explained by the rounding off of the link travel times in flow propagation Eqn. (11).

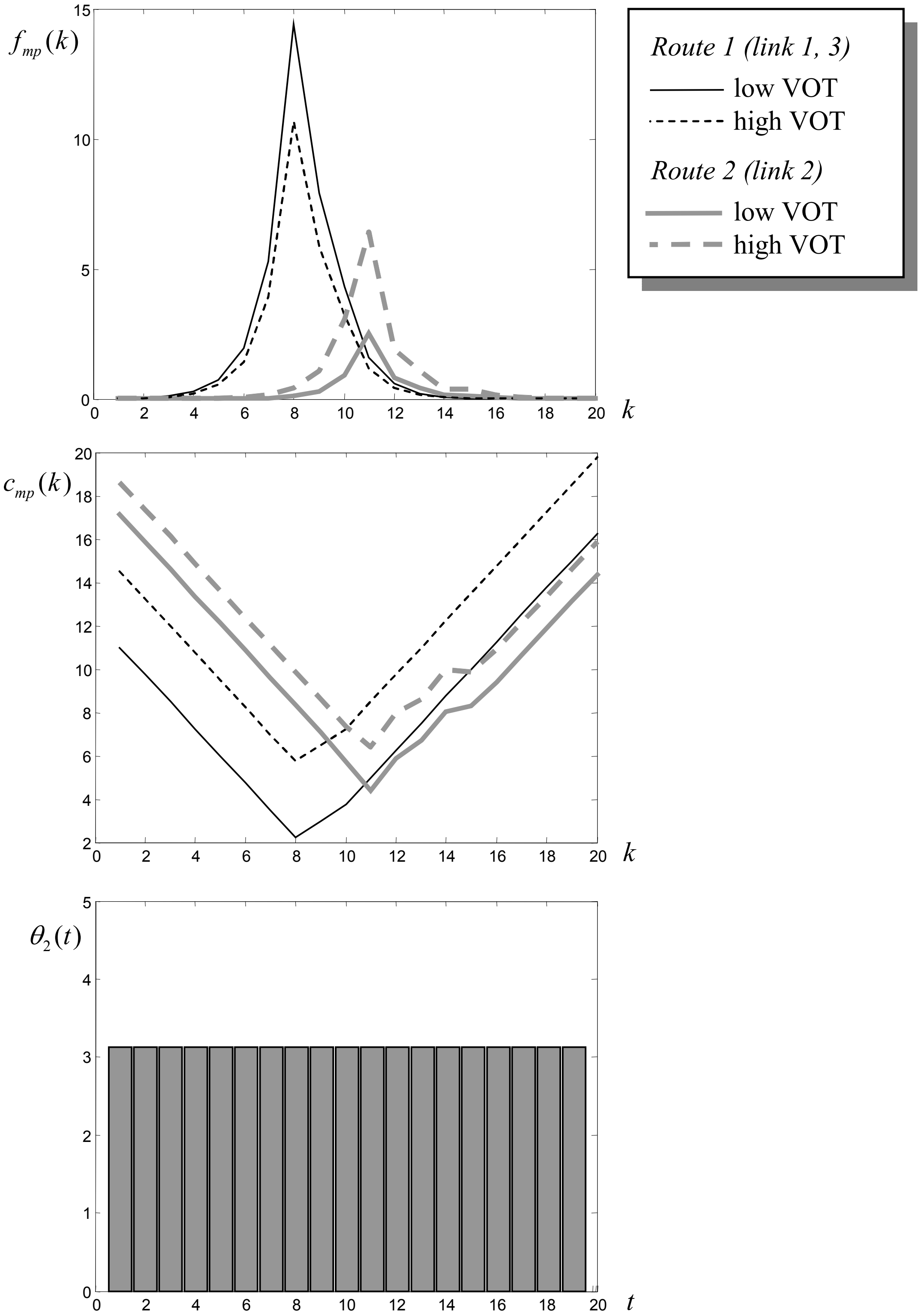

Figure 6: Route flows, costs, and toll levels when maximizing revenues

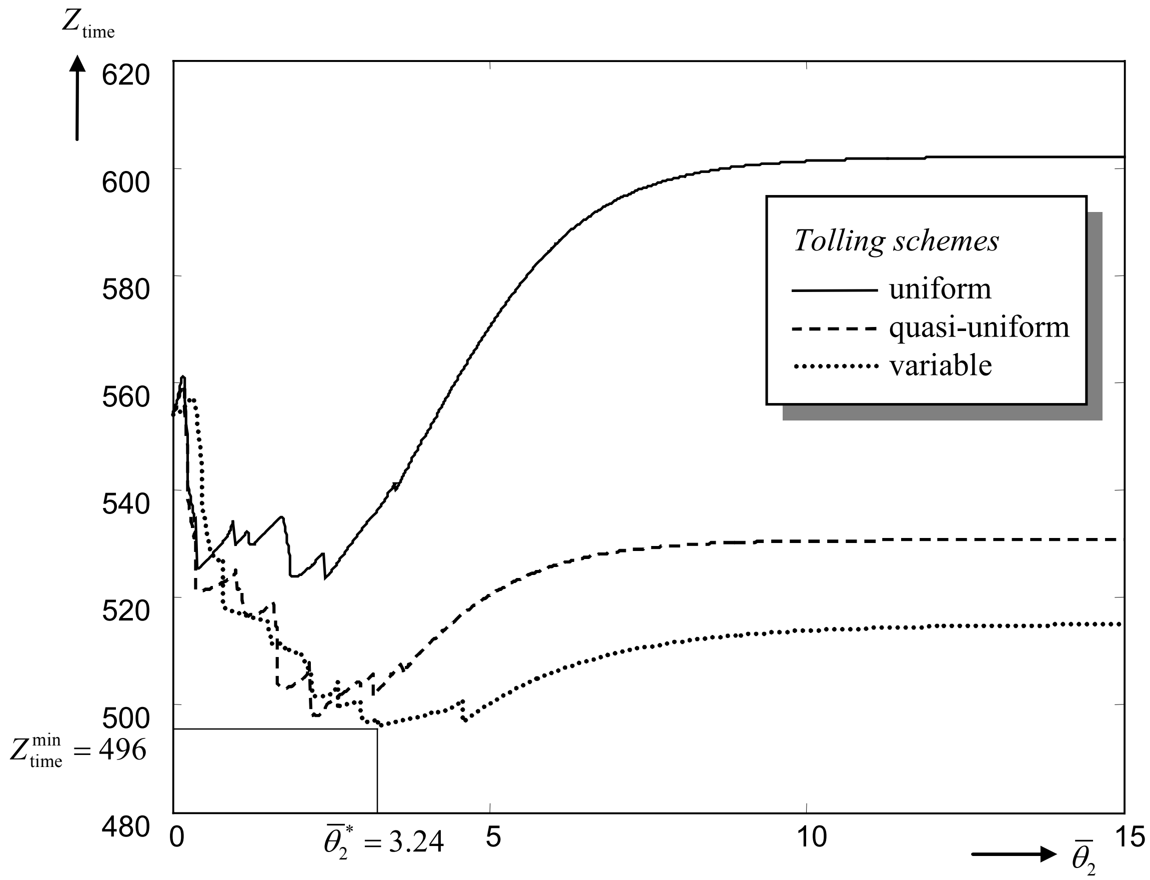

Figure 7: Total travel time for different tolling schemes and toll levels

The route costs and flows are depicted in Figure 8, together with the optimal toll levels for the objective of minimizing total travel time. Compared to Figure 6, it can be clearly seen that there are more departure time changes due to the fact that only the peak period is tolled, leading to a better spread of traffic over space and time and therefore lower total travel time.

4.4 Discussion

Results of both objectives (maximizing total toll revenues and minimizing total travel time) where different tolling schemes (uniform, quasi-uniform and variable) are applied are given in Table 1.

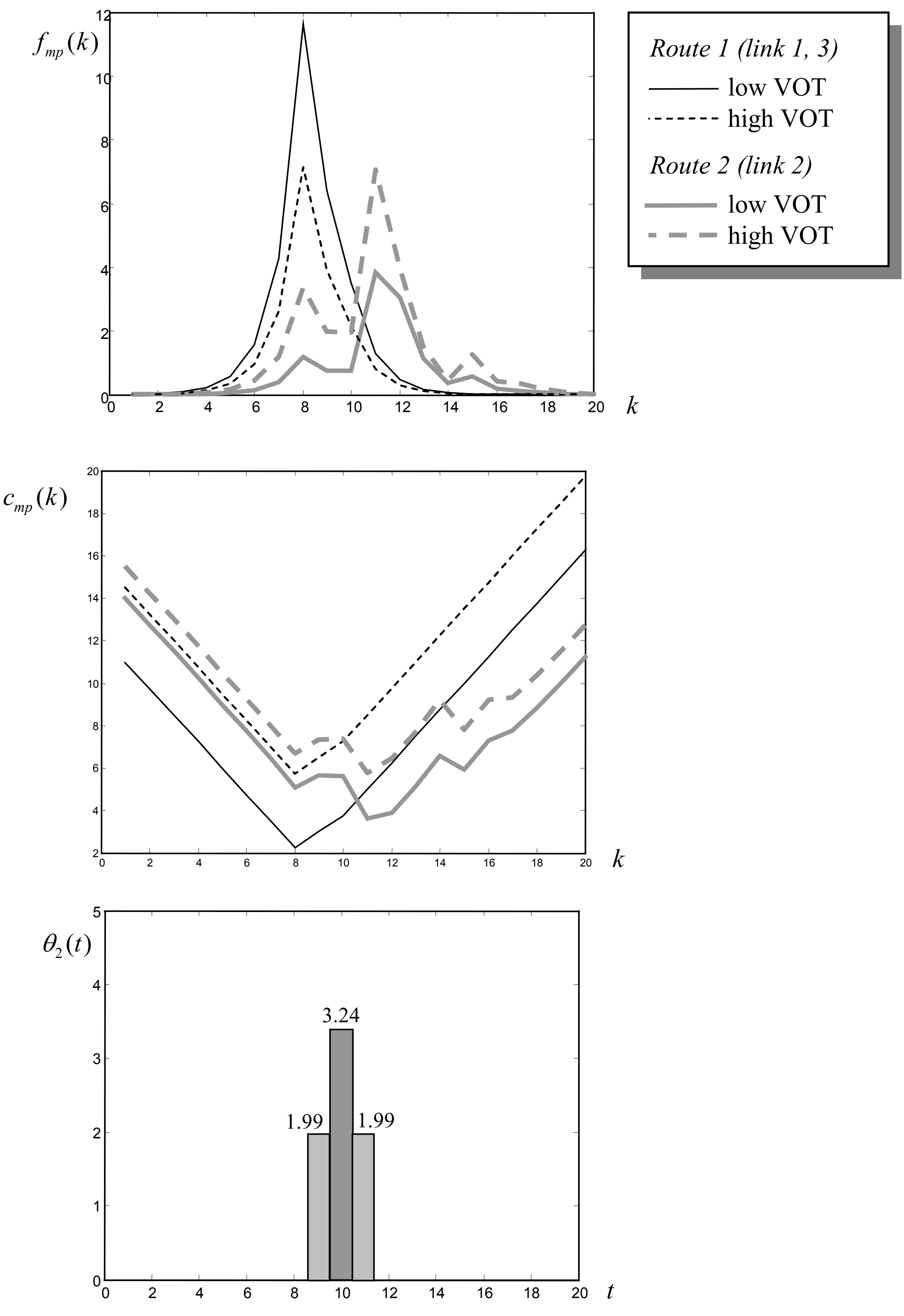

Figure 8: Route flows, costs and toll levels when minimizing total travel time

Table 1: Comparison of toll revenues and total travel time for different objectives

Objective: maximize total toll revenue			
Tolling scheme	*Optimal toll*	*Total revenue*	*Total travel time*
Uniform	3.11	**64.15**	534.48
Quasi-uniform	2.82	49.50	503.42
Variable	4.04	37.13	498.26
Objective: minimize total travel time			
Tolling scheme	*Optimal toll*	*Total revenue*	*Total travel time*
Uniform	2.41	60.81	523.56
Quasi-uniform	2.27	45.56	497.91
Variable	3.24	35.25	**496.02**

These results show that in the case of maximizing total toll revenues the best tolling scheme is uniform with toll level $\overline{\theta}_2 = 3.11$. However, this toll will yield a high total travel time (534.48). On the other hand, in the case of minimizing total travel time, the variable tolling scheme with $\overline{\theta}_2 = 3.24$ performs best. However, this toll will yield a low total toll revenue (35.25). In other words, maximizing total toll revenues and minimizing total travel time are opposite objectives. This can be explained as follows. In maximizing toll revenues, the road authority would like to have as many as possible travelers on the tolled route, hence trying to push as few as possible travelers away from the tolled alternative by imposing toll. In contrast, when minimizing total travel time, the road authority would like to spread the traffic as much as possible in time and space, hence trying to influence as many travelers as possible to choose other departure times and routes. Using a uniform tolling scheme, travelers are not changing their departure times, making it suitable for maximizing revenues, while in the variable tolling scheme other departure times are good alternatives, making it suitable for minimizing travel time. In any case, depending on the objectives of the road authority, there are different optimal tolling schemes with different toll levels.

5 Conclusions

A mathematical bi-level optimization problem has been formulated for the optimal toll network design problem. The road authority has some policy objectives, which they may optimize by imposing tolls. Second-best scenarios are considered in this paper, assuming that only a subset of links can be tolled. Different tolling schemes can be selected by the road authority, such as (quasi-)uniform and variable tolling schemes, each having a different impact on the policy objective. Due to tolls, the travelers may change their route and departure times. Heterogeneous travelers with high and low value of time are considered.

The aim of the research is to investigate the feasibility of the dynamic model framework proposed in this paper and to investigate properties of the objective function for different objectives and tolling schemes. The complex optimization problem has been solved using a simple grid search method, but for more practical case studies more sophisticated algorithms will be developed in the future.

In the case studies in the paper is shown that policy objectives can indeed be optimized by imposing tolls, and that different policy objectives lead to different optimal tolling schemes and toll levels.

Acknowledgements

We are grateful to the NWO Multi-disciplinary Pricing in Transport (MD-PIT) research program and the BSIK Next Generation Infrastructure (NGI) research program for their financial support.

References

Abou-Zeid, M. (2003). *Models and Algorithms for the Optimization of Traffic Flows and Emissions using Dynamic Routing and Pricing*. M.Sc. Thesis, Massachusetts Institute of Technology, Cambridge, MA, USA.

Beckmann M.J., C.B. McGuire, and C.B. Winsten (1956). *Studies in the Economics of Transportation*. Yale University Press, New Haven. CT, USA.

Ben-Akiva, M., A. De Palma, and P. Kanaroglou (1986). Dynamic Model of Peak Period traffic congestion with Elastic Arrival Rates. *Transportation Science 20(2)*, pp. 164-181.

Bliemer, M.C.J. and P.H.L. Bovy (2003) Quasi-Variational Inequality Formulation of the Multiclass Dynamic Traffic Assignment Problem. *Transportation Research B 37*, pp. 501-519.

Chabini, I.(2000). The analytical network loading problem: formulation, solution algorithms and computer implementations. *Transportation Research Record 1771*, pp. 191-200.

Dafermos S. (1973). Toll patterns for multiclass-user transportation networks. *Transportation Science 7*, pp. 211-223.

Joksimovic D., M.C.J. Bliemer and P.H.L. Bovy (2005). Optimal Toll Design Problem in Dynamic Traffic Networks- with Joint Route and Departure Time Choice. Accepted for publication in *Transportation Research Record*.

Mahamssani H.S. and R.Herman (1984). Dynamic User Equilibrium Departure time and route choice in Idealized Traffic Arterials. *Transportation Science 18 (4*), pp. 362-384.

May A. D. and D.S. Milne (2000). 'Effects of Alternative Road Pricing Systems on Network Performance'. *Transportation Research A 34(6)*, pp. 407-436.

Pigou A.C (1920). *Welth and welfare*. Macmillan, London, UK.

Ran, B. and D. E. Boyce (1996). *Modeling dynamic transportation networks: an intelligent transportation system oriented approach*, Springer-Verlag, 2nd edition, Berlin, Germany.

Verhoef E.T. (2002). Second-best Congestion Pricing in General Static Transportation Networks with Elastic Demand. *Regional Science and Urban Economics Journal 32*, pp. 281-310.

Viti, F., S.F. Catalano, M. Li, C.D.R. Lindveld, and H.J. van Zuylen (2003). An optimization Problem with dynamic route-Departure time choice and Pricing. In proceedings of 82nd *Transportation Research Board*, CD-ROM, Washington, D.C., USA.

Wie, B., and R.L. Tobin (1998). Dynamic Congestion Pricing Models for General Traffic Networks. *Transportation Record B 32(5)*, pp.313-327.

Yang H. and Huang H.-J (1998). Principle of marginal-cost pricing: how does it work in a general road network? *Transportation Research A 32(1)*, pp. 45-54.

Yang, H. (1999). System optimum, stochastic user equilibrium and optimal link tolls. *Transportation Science 33*, pp. 354-360.