DIFFERENT POLICY OBJECTIVES IN ROAD PRICING:
A GAME THEORY APPROACH

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Samenvatting

Verschillende Doeleinden in Prijsbeleid: Een Speltheoretische Benadering

In dit artikel richten we ons op een nieuwe benadering voor het formuleren van het probleem van optimaal prijsbeleid. Het doel is om meer inzicht te krijgen in het bepalen van optimale tolheffingen en het gedrag van de reizigers. Nutstheorie wordt gebruikt om het probleem te formuleren voor verschillende doelstellingen van de wegbeheerder. Diverse speltheoretische concepten worden gedefinieerd afhankelijk van de interactie tussen spelers (reizigers aan de ene kant en de wegbeheerder aan de andere kant). De wegbeheerder beïnvloedt de reizigiers door middel van tolheffingen, terwijl de reizigers reageren door hun reisgedrag aan te passen. De reizigers kunnen beslissen hun route te veranderen (om de tol te ontlopen), of kunnen beslissen om af te zien van de rit. Een spel tussen de spelers kan worden opgelost voor verschillende beleidsdoelstellingen van de wegbeheerder, namelijk: maximalisatie van totale nut, maximalisatie van de tolopbrengsten, en maximalisatie van het sociale surplus. Het spelconcept waarin de wegbeheerder een grotere marktinvloed heeft dan de reizigers het meest realistisch. Ter illustratie worden verschillende spellen met verschillende beleidsdoelstellingen gepresenteerd, waarbij wordt aangetoond dat verschillende doelstellingen leiden tot verschillende optimale tolheffingen.

Summary

Different Policy Objectives in Road Pricing: A Game Theory Approach

In this paper we focused on a new approach to formulate and solve optimal toll problem. The aim is to gain more insight into determining optimal tolls as well as into the behaviour of users. To formulate the problem we use utility maximisation theory including different objectives of the road authority. We define different game concepts dependent on the interaction between players (travellers on the one side and the road authority on the other). The road authority influences the travellers with different toll values while travellers react on these tolls by changing their travel behaviour. Travellers can decide to change their route (in order to avoid to pay tolls), or even decide not to travel at all. The games are solved for different policy objectives of the road authority: maximize total utility, maximize toll revenues and maximize social surplus. The game concept in which the road authority has more market power than the travellers is the most realistic game. An illustration of the games for different policy objectives is presented showing different optimal toll patterns for different objectives.

1. Introduction and background

The view that pricing can be one of the strategies to achieve more efficient use of the transportation system has led to expectation that road pricing can improve the use of transportation system. Many researches studied the effects of introducing road-pricing measures on transportation networks (e.g. May and Milne, 2000).

Road pricing is a very complex issue making it necessary to consider road pricing from different perspectives and approaches. In this paper we would like to answer the following questions:

- Which objectives the road authority would like to achieve?
- Who is involved in decision-making in the road pricing problem and how decision should be made?
- How will the travellers change their travel behaviour after introducing road-pricing problem?
- How will travellers interact with each other and how can the road authority influence (or even control) behaviour of travellers.
- What are the outcomes of different strategies of road authorities?

To answer such questions we need to establish a structural framework to analyse the behaviour of travellers as well as the road authority. Game theory provides such a frameworks for modelling decision-making processes in which multiple players are involved. Players have different objectives, rules of the game, different strategies and assumptions. In this paper we investigate interaction between the road authority and the travellers, the nature and the consequences of that interaction. The paper shows that depend on which objective the road authority will apply will have a strong influence on how, where, when and how much toll will be levied. Therefore, the focus of this paper is on assessing different objectives that the road authority may adopt and on their influence on optimal toll design.

In this paper we analyse in a game-theoretic framework a very simple route choice problem with elastic demand where road pricing is introduced. First, the road pricing is formulated using game theory notions with which different games are described. After that, a game-theoretic approach is applied to formulate the road pricing game as social planner (monopoly), Stackelberg and Cournot games, respectively. The main purpose of the experiments reported here is to show the outcomes of different games established for the optimal toll design problem.

2. Literature review

Game theory appeared for the first time in transportation field in the form of so-called *Wardropian equilibrium* of route choice ('no any traveller can be better of by unilaterally changing of routes'). For more information see (Wardrop, 1952). Wardrop equilibrium of route choice is similar to the Nash equilibrium of an N-player game (see, Nash, 1950). In (Fisk, 1984) for the first time different problems in transportation systems modelling are described using game theory approach. In that paper, different solutions algorithms for transportation are proposed. Relationships are drown between two game theory models based on the Nash non-cooperative and Stackelberg games.

The integrated traffic control and assignment problem is presented as a non-cooperative game between the road authority and highway users in the work of (Chen and Ben-Akiva, 1998). The objective of the combined control-assignment problem is to find dynamic system optimal settings and dynamic user-optimal flows. The combined control-assignment problem is first formulated as a single-level Cournot game: the traffic authority and the users choose their strategies simultaneously. Then, the combined problem is formulated as a bi-level Stackelberg game in which the traffic authority is the leader who determines the signal settings in anticipation of the user's responses.

The problem of determining optimal tolls in transportation networks is a complex issue. In (Levinson, 1988) the question what happens when jurisdictions have the opportunity to establish tollbooths at the frontier separating them is examined. If one jurisdiction would be able to set his policy in a vacuum it is clearly advantageous to impose as high toll on non-residents as can be supported. However, the neighbouring jurisdiction can set a policy in response. This establishes the potentional for a classical prisoner's dilemma consideration: in this case to tax (cooperate) or to toll (defect).

In (Levinson, 2003) an application of game theory and queuing analysis to develop micro-formulations of congestion can be found. Only departure time is analysed in the context of the two-player and three-player game respectively. Interactions among the players affect the payoffs for other players in a systematic way.

In (Joksimovic et. al., 2004) route choice and elastic demand is considered with focus on different game concepts between the road authority and the travellers, in the optimal toll problem. A few experiments are done showing that the Stackleberg game (the game where the road authority is the 'leader', and the travellers are 'followers') is the most promising game.

There is a lack in the literature about the importance of different policies the road authority may adopt, as well as the outcomes that can be result of the different objectives and games played with the travellers. Therefore, different policy objectives of the road authority in the optimal toll design problem will be the focus of this paper. Different game concepts will be investigated showing how well the road authority and travellers are performing.

3. Problem statement using non-cooperative game theory

The interaction between travellers and the road authority can be seen as a non-cooperative (N+1) players game between the road authority on the one side and the travellers in the other. The objective of the road-pricing problem is to find system-optimal tolls and user-optimal traffic flows simultaneously. The road-pricing problem is an example of *two-stage game*.

The user equilibrium traffic assignment problem (lower level problem) can be formulated as non-cooperative, N-person game and solved as a Nash game. The upper level problem may have different objectives depending on what the road authority would like to achieve. This issue will be the focus of this paper.

A conceptual framework for the road pricing problem in case of elastic demand addressed from different road authority's objectives is given in Figure 1.

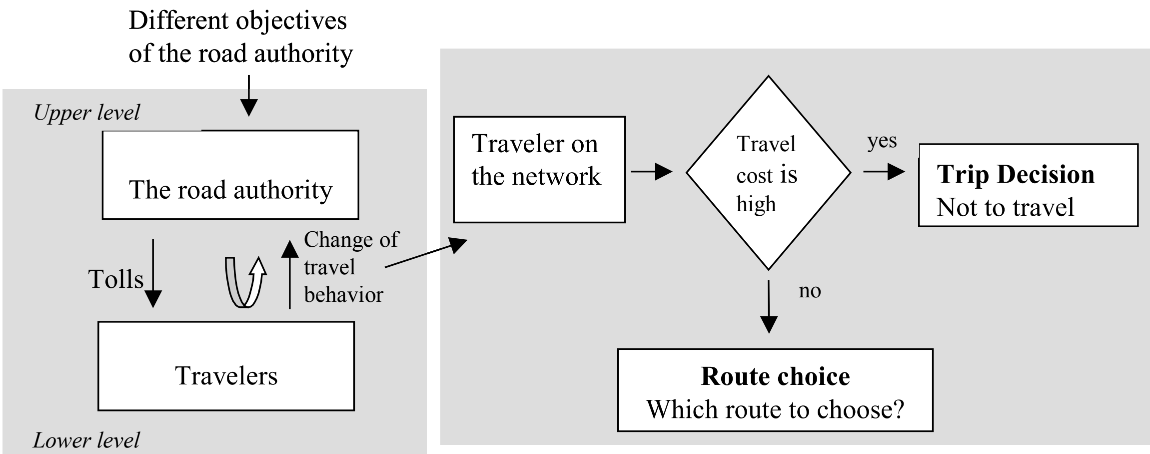

Figure 1: Conceptual framework for road-pricing with route and elastic demand

The road authority set tolls on the network while travellers respond to tolls by changing their travel decisions. Depending on the travel costs, they can decide to travel along a certain route or decide not to travel at all.

In the road-pricing problem, we are dealing with an *N*+1-player game, where there are *N* players (travelers) making a travel choice decision, and one player (the road manager) making a control or design decision (in this case, setting road tolls). Adding the traffic authority to the game is not as simple as extending an *N*-player game to an *N*+1 player game, because the strategy space and the payoff function for this additional player differs from the rest of the *N* players. In fact, there are two games played in conjunction with each other. The first game is a non-cooperative game where all *N* travelers aim to maximize their individual utility by choosing the best travel strategy (i.e. trip choice and route choice), taking into account all other travelers' strategies. The second game is between the travelers and the road manager, where the road manager aims to maximize some network performance by choosing a control strategy, taking into account that travelers respond to the control strategy by adapting their travel strategies. The two games can be described as follows:

The outer level game, being the toll design problem, consisting of the following elements:

1. Players: the authority on the one side and *N* potential travelers on the other;
2. Rule 1: the authority sets the tolls taking the travelers' behavior into account as well as possible restrictions on the toll levels in order to optimize a certain objective.
3. Rule 2: travelers react on travel costs (including tolls) and change their behavior (route choice, trip choice) as to maximize their individual subjective utilities.
4. Outcomes of the game: a) optimal strategies for the authority (tolls), b) payoff for the authority (e.g. social welfare, revenues), c) optimal strategies for the travelers (trip and route decisions) and d) payoff for the travelers (utilities). The outcomes depend on the objective function for the authority used in the model.

The inner level game, being the network equilibrium problem, consisting of the following elements:

1. Players: *N* travelers
2. Rule: travelers make optimal trip and route choice decisions as to maximize their individual subjective utilities given a specific toll pattern.
3. Outcome of the game: a) optimal strategies for the travelers (trip and route decisions), b) payoff for the travelers (utilities)

Our main focus in this paper is to investigate the outer level game between the road authority and users, although the inner level game between travelers is part of it.

4. Model structure

The objectives of the road authority and the travelers are different and sometimes even opposite. The upper level objective may be to minimize total travel time, to relieve congestion, to improve safety, to raise revenue, to improve total system utility, or anything else. The lower level objective may be the individual travel time, travel cost, or the individual travel utility. In this paper, we use the individual travel utility as the objective to maximize for travelers.

Since the purpose of this paper is to gain more insight into the structure of the optimal toll design problem under different policy objectives by using game theory, we restrict ourselves to the case of a very simple network in which only one origin-destination (OD) pair is considered. Between this OD pair, different non-overlapping route alternatives are available. The generalized route travel cost function, c_{pi} of traveler *i* for route *p* includes the travel time costs and the toll costs,

$$c_{pi} = \alpha\tau_p + \theta_p, \tag{1}$$

where τ_p is the travel time of route *p*, θ_p is the toll costs of route *p*, and α denotes the value-of-time (VOT) which converts the travel time into monetary costs. Let U_{pi} denote the trip utility for making a trip along route *p* of traveler *i*. This trip utility consists of a fixed net utility $\bar{U}$ for making the trip (or arriving at the destination), and a disutility consisting of the generalized route travel costs c_{pi}:

$$U_{pi} = \bar{U} - c_{pi}. \tag{2}$$

According to utility maximization theory, a trip will be made only if the utility of doing an activity at a destination minus the utility of staying at home and the disutility of traveling is positive. In other words, if $U_p \leq 0$ then no trip will be made. By including a *fictitious* route in the route choice set representing the travelers' choice not to travel, and attaching a disutility of zero to this 'route' alternative, we combine route choice and trip choice into the model. Travelers are assumed to respond according to Wardrop's equilibrium law extended with elastic demand: *At equilibrium, no user can improve its trip utility by unilaterally making another route choice or trip choice decision.*

For the sake of simplicity we assume the deterministic utility case without a random error term. For more elaborate definitions, see (Cascetta, 2001).

5. Game theory applied to road pricing

Let us consider first the *N*-player game of the travelers, where S_i is the set of available alternatives for traveler *i*, $i \in \{1,\ldots,N\}$. The strategy $s_i \in S_i$ that traveler *i* will play depends on the control strategy set by the road manager, denoted by vector θ, and on the strategies of all other players, denoted by $s_{-i} \equiv (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_N)$. We assume that each traveler decides independently seeking unilaterally the maximum utility payoff, taking into account the possible rational choices of the other travelers. Let $J_i\left(s_{-i}(\theta),s_i(\theta),\theta\right)$ denote the utility payoff for traveler *i* for a given control strategy θ. This utility payoff can include all kinds of travel utilities and travel cost. Utility payoff for traveler *i* can be expressed as follows:

$$J_i\left(s_{-i}\left(\theta\right),s_i\left(\theta\right),\theta\right)=U_{pi}-c_{pi} \tag{3}$$

where c_{pi} is defined in expression (1) and U_{pi} in expression (2).

If all other travelers play strategies s_{-i}^*, then traveler *i* will play the strategy that maximizes his payoff utility, i.e.

$$s_i^*(\theta)=\arg\max_{s_i\in S_i} J_i\left(s_{-i}^*(\theta),s_i(\theta),\theta\right). \tag{4}$$

If Equation (4) holds for all travelers $i \in \{1,\ldots,N\}$, then $s^*(\theta)\equiv(s_{-i}^*(\theta),s_i^*(\theta))$ is called a *Nash equilibrium* for the control strategy θ. In this equilibrium, no traveler can improve his utility payoff by unilateral change of behavior. Note that this coincides with the concept of a *Wardrop user-equilibrium*.

Now consider the complete *N*+1-player game where the road manager faces the *N* travelers. The set Θ describes the alternative strategies available to the road manager. Suppose he chooses strategy $\theta \in \Theta.$, then, depending on this strategy and on the strategies $s^*(\theta)$, chosen by the travelers, his utility payoff is $R(s^*(\theta),\theta)$, and may represent e.g. the total system utility or the total profits made. The road manager chooses the strategy θ^* in which he aims to maximize his utility payoff, depending on the responses of the travelers:

$$\theta^* = \arg\max_{\theta\in\Theta} R\left(s^*(\theta),\theta\right). \qquad (5)$$

If Equations (4) and (5) are satisfied for all (N+1) players, where $\theta = \theta^*$ in Equation (4), then this is a Nash equilibrium in which no player can be better off by unilaterally following another strategy. Although all equilibria use the Nash concept, a different equilibrium or game type can be defined in the N+1-player game depending on the influence each of the players has in the game. Game theory notions used in this paper are adopted from work of (Altman at al., 2003).

6. Different game concepts

In the following we will distinguish three different types of games between the road authority and the travelers, namely, Monopoly, Stackelberg and Cournot game, respectively.

6.1 Social planner game

In this case, the road manager not only sets its own control, but is also assumed able to control the strategies that the travelers will play. In other words, the road manager sets θ^* as well as s^*. This case will lead to a so-called system optimum solution of the game. A Social planner (monopoly or solo player) game represents the best system performance and thus may serve as a 'benchmark' for other solutions. This game solution shows what is best for the one player (the road manager), regardless of the other players. In reality, however, a social planner solution may not be realistic since it is usually not in the users' best interest and is it practically difficult to force travelers choosing a specific route without an incentive. From an economic point of view, in this case the road authority has complete (or full) market power. Mathematically, the problem can be formulated as follows:

$$(s^*,\theta^*) = \arg\max_{\theta\in\Theta,\, s\in S} R(s,\theta). \qquad (6)$$

6.2 Stackelberg game

In this case, the road manager is the 'leader' by setting the control, thereby directly influencing the travelers that are considered to be 'followers'. The travelers may only indirectly influence the road manager by making travel decisions based on the control. It is assumed that the road manager has complete knowledge of how travelers respond to control measures. The road manager sets θ^* and the travelers follow by playing $s^*(\theta^*)$. From an economic point of view, in a Stackelberg game one player has more market power than others players in the game (in this case the road authority has more market power than the travelers). The Stackelberg game is a *dynamic*, multi-stage game of complete and perfect information. The order in which decisions are made is important. In a game with complete information, every player is fully informed about the rules of the game, the preferences of each player, and each player knows that every player knows. In other words, for the road pricing game we assume that apart from the rules of the game all information about travel attributes (toll levels, available routes, travel times) are known to all travelers. A game with perfect information means that decision makers know the entire history of the game.

For more details about complete and imperfect games, see e.g. (Ritzberger, 2002). The equilibrium is determined by *backward induction* where the traffic authority initiates the moves by setting a control strategy. Steps for the road pricing game are as follows:

1. The road authority chooses toll values from the feasible set of tolls.
2. The travelers react on the route cost (with tolls included) by adapting their route and/or trip choice.
3. Payoffs for the road authority as well as travelers are computed.
4. The optimal strategy for the road authority including the strategies of travelers is chosen.

The problem can be mathematically formulated as to find (s^*, θ^*) such that:

$$\theta^* = \arg\max_{\theta \in \Theta} R(s^*(\theta), \theta), \quad \text{where} \quad s_i^*(\theta) = \arg\max_{s_i \in S_i} J_i(s_i, s_{-i}^*, \theta), \quad \forall i = 1, \ldots, N. \qquad (7)$$

6.3 Cournot game

In contrast to the Stackelberg game, in this case the travelers are now assumed to have a direct influence on the road manager, having complete knowledge of the responses of the road manager to their travel decisions. The road manager sets $\theta^*(s^*)$, depending on the travelers' strategies $s^*(\theta^*)$. This type of a so-called duopoly game, in which two players choose their strategies simultaneously and therefore one's player's response is unknown in advance to others, is known as a Cournot game. Mathematically the problem can be formulated as follows. Find (s^*, θ^*) such that

$$\theta^* = \arg\max_{\theta\in\Theta} R(s_i^*, s_{-i}^*, \theta), \quad \text{and} \quad s_i^* = \arg\max_{s_i\in S_i} J_i(s_i, s_{-i}^*, \theta^*), \quad \forall i = 1,\ldots,N. \tag{8}$$

The different game concepts will be illustrated in the next section. It should be pointed out that the Stackelberg game is the most realistic game approach in our pricing context. This is a dynamic game which can be solved using backward induction, see e.g. (Basar and Olsder, 1995). Mathematical bi-level problem formulations can be used for solving more complex games, see e.g. *(*Joksimovic et al., 2005*)*.

7. Different objectives of the road authority

Which objective the road authority will apply will have influence on the optimal toll levels. Depending on the authority's objective, different utility payoff functions can be formulated.

Assuming the road authority's objective of maximizing total travel utility (the utility of all network users together), the objective is defined as the sum of the payoff values of all travelers:

$$\max R\left(s^*(\theta),\theta\right) = \sum_{i=1}^{N} J_i(s^*(\theta)). \tag{9}$$

In case the road authority aims at maximizing total toll revenues, the following objective may be used:

$$\max R\left(s^*(\theta),\theta\right) = \sum_{p} q_p(s^*(\theta))\theta_p, \tag{10}$$

where $q_p(s^*)$ denotes the number of travelers using route *p*, which can be derived from the optimal strategies s^*. Clearly, setting tolls equal to zero does not provide any revenues, while setting very high tolls will make all travelers decide not to travel at all.

Combining these two objectives leads to the notion of social surplus maximization. The social surplus can be computed by adding the toll revenues to the total trip utilities, such that the following problem will maximize social surplus as an objective:

$$\max R\left(s^*(\theta),\theta\right) = \sum_{i=1}^{N} J_i(s^*(\theta)) + \sum_{p} q_p(s^*(\theta))\theta_p. \tag{11}$$

8. A few experiments

Let us now look at the following simple problem to illustrate how the road-pricing problem can be analyzed using game theory. Suppose there are two individuals wanting to travel from A to B. There are two alternative routes available to go to *B*. The first route is tolled (toll is equal to θ), the second route is untolled. Depending on the toll level, the travelers decide to take either route 1 or route 2, or not to travel at all. The latter choice is represented by a third virtual route, such that we can consider three route alternatives as available strategies to each traveler, i.e. $S_i = \{1,2,3\}$ for traveler i = 1,2. Figure 2 illustrates the problem.

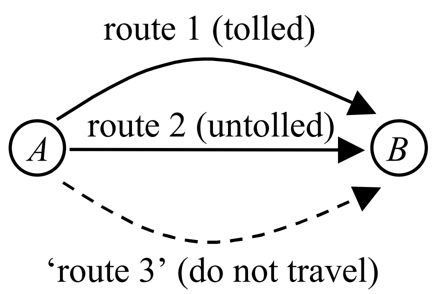

Figure 2 Network description for the road- pricing problem

Each strategy yields a different payoff, depending on the utility to make the trip, the travel time on the route (that increases whenever more travelers use it) and a possible route toll. We assume that traveler *i* aims to maximize its individual travel utility (payoff,) given by

$$J_i(s_1(\theta), s_2(\theta)) = \begin{cases} \bar{U} - \alpha\tau_1(s_1(\theta), s_2(\theta)) - \theta, & \text{if } s_i(\theta) = 1, \\ \bar{U} - \alpha\tau_2(s_1(\theta), s_2(\theta)), & \text{if } s_i(\theta) = 2, \\ 0, & \text{if } s_i(\theta) = 3. \end{cases} \qquad (12)$$

In Equation (5), $\bar{U}$ represents the trip utility when making the trip to destination *B* (in the calculations we assume $\bar{U}$ =210), $\tau_p^{rs}(\cdot)$ denotes the route travel time for route *r* depending on the chosen strategies, while α represents the value of time (we assume $\alpha = 6$ for all travelers). Note that negative net utilities on route 1 and 2 imply that one will not travel, i.e. if the cost (disutility) of making the trip is larger than the utility of the trip itself. The route travel times are given as a function of the chosen strategies in the sense that the more travelers use a certain route, the higher the travel time:

$$\tau_1(s_1(\theta), s_2(\theta)) = \begin{cases} 10, & \text{if } s_1(\theta) = 1 \text{ or } s_2(\theta) = 1 \text{ (e.g. flow on route 1 is 1)}, \\ 18, & \text{if } s_1(\theta) = 1 \text{ and } s_2(\theta) = 1 \text{ (e.g. flow on route 1 is 2)}, \end{cases} \tag{13}$$

and

$$\tau_2(s_1(\theta), s_2(\theta)) = \begin{cases} 20, & \text{if } s_1(\theta) = 2 \text{ or } s_2(\theta) = 2 \text{ (e.g. flow on route 2 is 1)}, \\ 40, & \text{if } s_1(\theta) = 2 \text{ and } s_2(\theta) = 2 \text{ (e.g. flow on route 2 is 2)}. \end{cases} \tag{14}$$

Solving the game between the two travelers for a Nash equilibrium corresponds to a Wardrop equilibrium with elastic demand, in which no traveler can improve his/her utility by unilaterally changing route or deciding not to travel. For the sake of clarity we will only look at pure strategies in this example, but the case may be extended to mixed strategies as well. In pure strategies, each player is assumed to adopt only one strategy, whereas in mixed strategies, the players are assumed to adopt probabilities for choosing each of the available strategies. In our example we are thus looking at discrete flows instead of continuous flows so that. Wardrop's first principle according to which all travel utilities are equal for all used alternatives may no longer hold in this case. In fact, the more general equilibrium rule applies in which each traveler aims to maximize his personal trip utility. The utility payoff table, depending on the toll θ, is given in Table 1 for the two travelers, where the values between brackets are the payoffs for travelers 1 and 2, respectively.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$(102-\theta, 102-\theta)$	$(150-\theta, 90)$	$(150-\theta, 0)$
	Route 2	$(90, 150-\theta)$	$(-30, -30)$	$(90, 0)$
	Route 3	$(0, 150-\theta)$	$(0, 90)$	$(0, 0)$

Table 1 Utility Payoff Table for Travellers

For example, if traveler 1 chooses route 1 and traveler 2 chooses route 2, then the travel utility for traveler 1 is $J_1(1,2) = 210 - 6 \cdot 10 - \theta = 150 - \theta.$

In the experiments we will consider three different road authority's objectives: total travel utility, social surplus, and generating revenues. For the first objective, three different game concepts are applied: social planner, Stackelberg and Cournot game, respectively. Because

Stackelberg game is the most realistic game and, we apply only Stackelberg game for the other two objective functions.

8.1 Case Study 1: Maximimise total travel utility

Now, let us add the road manager as a player, assuming that he tries to maximize total travel utility, i.e.

$$\text{Max}\, R\left(s^*(\theta),\theta\right) = J_1(s^*(\theta)) + J_2(s^*(\theta)). \qquad (15)$$

The strategy set of the road manager is assumed to be $\Theta = \{\theta \mid \theta \geq 0\}$. The payoffs for the road manager are presented in Table 2 depending on the strategy $\theta \in \Theta$ that the road manager plays and depending on the strategies the travelers play.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$204-2\theta$	$240-\theta$	$150-\theta$
	Route 2	$240-\theta$	-60	90
	Route 3	$150-\theta$	90	0

Table 2 Utility Payoff Table for the Road Manager if his Objective is to Maximize the Total Travel Utility

Let us solve the previously defined payoff tables for different game concepts and different values of tolls. First, we discuss the social planner game, then the Stackelberg game and finally the Cournot game.

8.1.1 Social planner game

In the social planner game, the road manager sets the toll as well as the travel decisions of the travelers such that his payoff is maximized. Note that the travel utility always decreases as θ increases, hence $\theta^* = 0$. In this case, the maximum utility can be obtained if the travelers distribute themselves between routes 1 and 2, i.e. $s^* = \{(1,2),(2,1)\}$. Hence, in this system optimum, the total travel utility in the system is 240. Note that this optimum would not occur if travelers have free choice, since $\theta = 0$ yields a Nash-Wardrop equilibrium for both travelers to choose route 1.

8.1.2. Stackelberg game

Now the travelers will maximize individually their own travel utility, depending on the toll set by the road manager. Figure 3 illustrates the total travel utility for different values of θ with the corresponding optimal strategies played by the travellers. When $0 \le \theta < 12,$ travellers will both choose route 1. If $12 \le \theta < 150,$ travellers distribute themselves between route 1 and 2, while for $\theta \ge 150$ one traveller will take route 2 and another traveller will not travel at all. Clearly, the optimum for the road manager is $\theta^* = 12,$ yielding a total travel utility of 228.

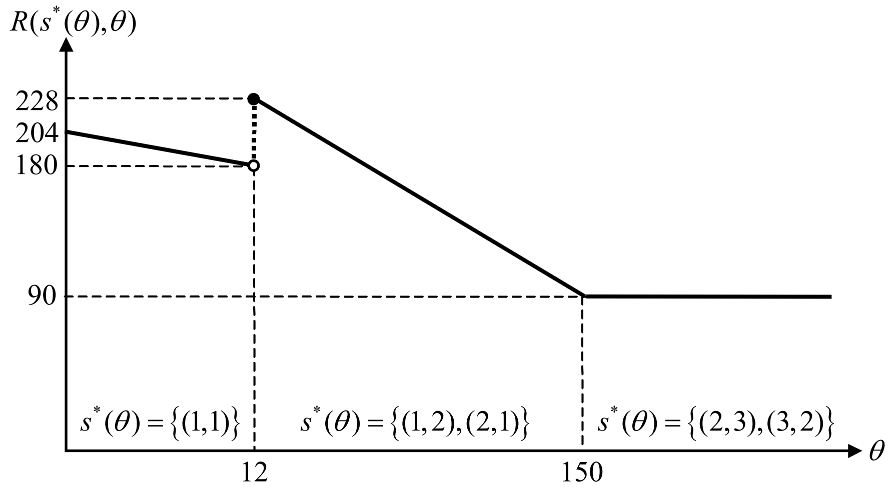

Figure 3 Total travel utilities depending on toll value for the objective of maximizing total travel utility

8.1.3. Cournot game

It can be shown that in case the travelers and the road manager have equal influence on each others strategies, multiple Cournot solutions exist. There is however one dominating strategy, being that the travelers both take route 1 and that the road manager sets zero tolls, yielding a total system utility of 204.

Table 3 summarizes the outcomes for the three different games.

Game	θ^*	$s_i^*(\theta)$	R	J_i
Social planner	0	$\{(1,2),(2,1)\}$	240	$\{(90,150),(150,90)\}$
Stackelberg	12	$\{(1,2),(2,1)\}$	228	$\{(90,138),(138,90)\}$
Cournot	0	$\{(1,1)\}$	204	$\{(102,102)\}$

Table 3 Comparison of Outcomes of Different Games for the Objective of Maximizing Total Travel Utility

8.2 Case study 2: Maximize social surplus

Now, the road manager is assumed to maximize social surplus (see formula (11)). The strategy set of the road manager is assumed to be $\Theta = \{\theta \mid \theta \geq 0\}$. The payoffs for the road manager are presented in Table 4 depending on the strategy $\theta \in \Theta$ that the road manager plays and depending on the strategies the travelers play.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	204	240	150
	Route 2	240	-60	90
	Route 3	150	90	0

Table 4 Utility Payoff Table for the Road Manager if his Objective is to Maximize Social Surplus

8.2.1 Stackelberg game

Now the travelers will maximize individually their own travel utility (see formula (9)). Figure 4 illustrates the social surplus for different values of θ with the corresponding optimal strategies played by the travelers. When $0 \leq \theta < 12$, travelers will both choose route 1. If $12 \leq \theta < 150$, travelers distribute themselves between route 1 and 2, while for $\theta \geq 150$ one traveler will take route 2 and another traveler will not travel at all. Clearly, the optimum for the road manager is $12 \leq \theta^* \leq 150$, yielding a total system utility of 240.

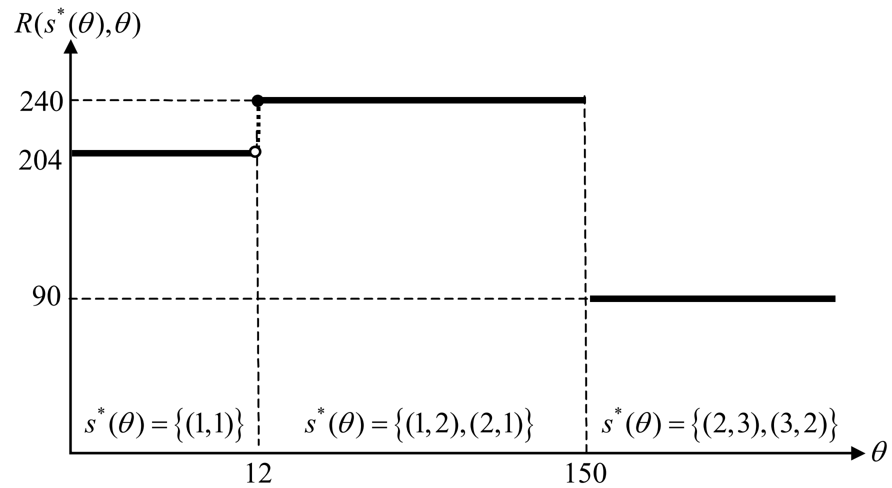

Figure 4 Total travel utilities depending on toll value for the objective of maximizing social surplus

8.3 Case study 3: Maximize revenues

Now, let us add the road manager as a player, assuming that he tries to maximize revenues (see formula (10)). The strategy set of the road manager is assumed to be $\Theta = \{\theta \mid \theta \geq 0\}$. Depending on the strategy $\theta \in \Theta$ that the road manager plays and depending on the strategies the travelers play, the payoffs for the road manager are presented in Table 5.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	2θ	θ	θ
	Route 2	θ	-60	90
	Route 3	θ	90	0

Table 5 Utility Payoff Table for the Road Manager if his Objective is to Maximize Revenues

8.3.1 Stackelberg game

Figure 5 illustrates the revenues for different values of θ with the corresponding optimal strategies played by the travelers. When $0 \leq \theta < 12$, travelers will both choose route 1. If $12 \leq \theta < 150$, travelers distribute themselves between route 1 and 2, while for $\theta \geq 150$ one traveler will take route 2 and another traveler will not travel at all. Clearly, the optimum for the road manager is $\theta^* = 150$, yielding a total system utility of 240.

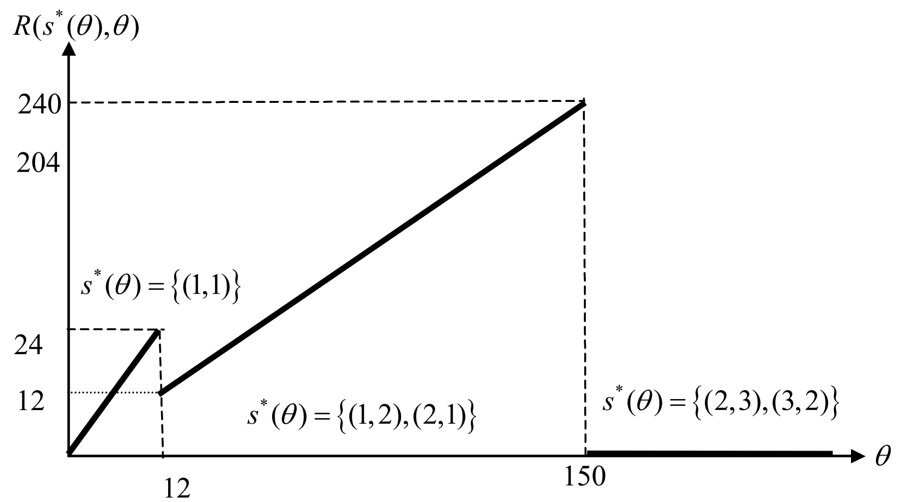

Figure 5 Total travel utilities depending on toll value for the objective of maximizing revenues

Considering all three case studies some conclusions can be drawn:

- There exist different objectives that all can be applied depending on what the road authority would like to achieve;
- Different objectives lead to different outcomes, both in terms of optimal toll system as well as in payoffs for players;
- Looking at different game types shows the span of outcomes of an optimal design and their relative worth;
- There exist multiple optimal solutions (multiple Nash equilibria)
- The objective function may have a non-continuous shape (jumps)

9. Conclusions and further extensions

The purpose of the paper was to gain more insight into the road-pricing problem using concepts from game theory as well as different toll designs depending on different objectives. To that end we presented the notions of game theory and presented three different game types in order to elucidate the essentials of the game theoretic approach. These game types were applied to three different toll design objectives exemplified on a simplistic demand-supply network system. This clearly revealed differences in design results in terms of toll levels and payoffs for involved actors, being the road authority and network users.

The theory presented here can be extended to include other relevant travel choices such as e.g. departure time choice as well as to include heterogeneous travellers and imperfect information on the part of the road users. An important extension is to apply the proposed game-theory framework to large cases (e.g. for large number of players or on bigger network). For practical use, the presented game-theoretic analysis should be translated into a modelling system with which tolling designs for real-size road networks become feasible. For that purpose, the bi-level optimisation framework will be used (see (Joksimovic et al., 2005)).

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