The Economics of Intersections: Understanding the Causal Mechanisms that Govern the Optimal Regulation of Intersections.

Ruth Evers
KU Leuven, Center for Economic Studies
ruth.evers@kuleuven.be

Stef Proost
KU Leuven, Center for Economic Studies
stef.proost@kuleuven.be

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Samenvatting

Economisch inzicht in de determinanten van de optimale regeling van kruispunten.

Kruispunten zijn een veelvoorkomende schakel in het verkeersnetwerk. Een goede regeling van het kruispunt is dan ook essentieel voor een efficiënt gebruik van de beschikbare capaciteit. De complexiteit van het probleem maakt de optimalisatie van kruispunten echter geen gemakkelijke opdracht. Deze paper reikt hiervoor een extra hulmiddel aan door, op basis van economische theorie, inzicht te verschaffen in de determinanten van een optimale regeling. Teneinde deze determinerende factoren te kunnen identificeren, optimaliseren we in deze paper de regeling van een kruispunt van twee wegen die eenzelfde herkomst-bestemmingspaar verbinden. Deze wegen kunnen zowel congestiegevoelig als relatief congestieongevoelig zijn. We bestuderen enerzijds het effect van voorrangsregels en anderzijds het effect van verkeerslichten. Op basis van een spel-theoretische analyse bekomen we een optimale oplossing voor dit simpele twee wegen-1 kruispunt-probleem. Meer bepaald tonen we aan dat voor een kruispunt waarop een voorrangsregeling van toepassing is, het optimale beleid er over het algemeen in bestaat om één van de twee wegen af te sluiten. Als het kruispunt geregeld wordt door verkeerslichten, kan het enkel optimaal zijn beide wegen te behouden als ze beiden congestiegevoelig zijn. De meerwaarde van deze resultaten is drievoudig. Ten eerste bevestigt het tegen-intuïtieve karakter van deze resultaten het belang van een goed inzicht in de determinanten van een optimale regeling. Ten tweede laat het -uit deze resultaten- verworven inzicht toe grotere netwerken op een efficientere en effectievere manier op te lossen. Efficienter omdat de oplossingstijd gereduceerd kan worden door een beter inzicht in de locatie van de optimale oplossing, effectiever omdat lokale optima gemakkelijker gedetecteerd kunnen worden. Ten slotte kunnen de bekomen resultaten ook zelf toegepast worden. In deze paper tonen we bijvoorbeeld aan hoe de bekomen resultaten de wetenschappelijke onderbouw kunnen bieden voor de keuze voor autoluw stadscentrum. We sluiten deze paper af met een uitbreiding van de analyse naar een casus met minimale groentijd, meer bepaald het probleem van sluipverkeer.

1. Introduction

Despite decades of research on the optimization of intersections, the poor regulation of intersections is a matter of huge frustration amongst many drivers today. The complexity of the problem makes the optimization of intersections a hard nut to crack. Understanding the causal mechanisms that govern optimal regulation of intersections is therefore essential in dealing with complex network problems. Not least because externalities, resulting from the selfish behavior of drivers, can lead to results that defy intuition.

In this paper, a hierarchical game theoretic framework is developed to model the strategic decisions of both the traffic authority and the drivers. In this so-called Stackelberg game the leader (traffic authority) moves first and bases their decisions on the expected reaction of follower (the drivers). To obtain an optimal and traceable solution we use a simple two-road intersection, that can be controlled by either a priority rule or traffic lights. This simplified network structure can represent different types of routes and different modes of transport that are either congestible or insensitive to congestion.

The remainder of this paper is organized as follows. After a literature review in Section 2, the intersection problem at hand and the assumptions are set out in Section 3. Section 4 presents the main results and Section 5 illustrates the theory of the previous section by means of two applications. Section 6, finally, offers a conclusion.

2. Literature review

The first traffic signal control model study was undertaken by Webster (1958). This model assumes traffic flows to be unaffected by the signal settings. This reduces the model to an isolated control problem in which signal settings are optimized for given flows on the network. The need to take into account the effects of the change in traffic light settings on the network flow was emphasized by Allsop (1974). Together with Charlesworth, Allsop developed an iterative convergent approach to solve the combined assignment and control problem in which the signal settings are optimized taking into account the assignment of the traffic flow according to the user equilibrium (Allsop and Charlesworth (1977)).

Fisk (1984) was the first to model the combined assignment and control problem as a Stackelberg game. Chen and Ben-Akiva (1998) developed a dynamic model dealing with the combined assignment and control problem and formulated it as a Cournot, Stackelberg and monopoly game. Overall, there have only been a limited number of authors following Fisk's example in modeling the combined assignment and control problem as a Stackelberg game. For an overview, see Hollander and Prashker (2006).

3. Problem formulation and methodology

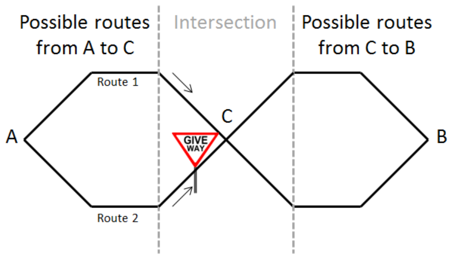
The first subsection introduces the reader to the Stackelberg game and Nash equilibrium. The following subsections elaborate the basic assumptions taken in this paper.

3.1 The Stackelberg game

The basic Stackelberg game can be described as follows. A leader chooses his strategy $u_L \in D_L \subseteq \mathbb{R}^{n_L}$ after which the follower determines his optimal response $u_F \in D_F \subseteq \mathbb{R}^{n_F}$. So, u_F is a function of u_L and this relation is determined by $\min_{u_F} O_F\,(u_L, u_F)$ with O_F the objective function of the follower. Before making his decision the leader will take into account this behavior of the follower and hence will choose u_L such as to minimize $O_L(u_L, f_F(u_L))$.
The behaviour of the drivers can also be represented as a game because the congestion on one road is dependent upon how many users choose to use the same road. In this paper, we will assume that the followers behave collectively according to the noncooperative principle of Nash which means that for each $u_L \in D_L$ they will choose a joint response vector $u_F^* \in G_F(G_F = \prod_{i=1}^{M} D_{Fi})$ such that for every follower *i*=1, ...,M and $\forall u_{Fi} \in D_{Fi}, u_{Fi} \neq u_{Fi}^* :\; O_{Fi}(u_L, u_{F1}^*, \dots, u_{Fi-1}^*, u_{Fi}^*, u_{Fi+1}^*, \dots, u_{FM}^*) \leq O_{Fi}(u_L, u_{F1}^*, \dots, u_{Fi-1}^*, u_{Fi}, u_{Fi+1}^*, \dots, u_{FM}^*)$.
In a two-road network, there are three possible user equilibria: users can all elect (or be forced) to take one of the two particular routes, or spread themselves between the two routes, dependent on the user cost. When both routes are used, the user cost of both routes has to be equal in equilibrium (Wardrop, 1952).

3.2 Assumptions underlying the priority-model

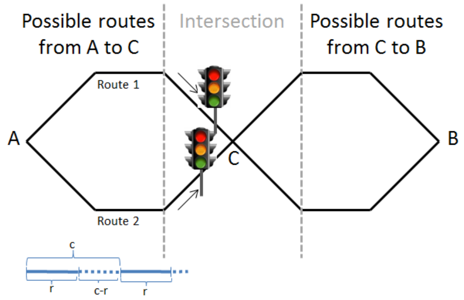
Figure 1 : Schematical outline of an intersection of two routes connecting one OD-pair (AB) regulated by a priority rule.



Three assumptions are imposed on the model representing the intersection regulated by a priority rule. The first assumption states that demand is inelastic and equals N. The second assumption ascertains that the users on route 2 always have to give way to those on route 1. And the third assumption postulates that v hours before a car on route 1 passes C, cars on route 2 already wait until the car on route 1 has passed. The final assumption states that all users are identical and aim to minimize their expected travel cost.

3.3 Assumptions underlying the traffic lights-model

Figure 2 : Schematical outline of an intersection of two routes connecting one OD-pair (AB) regulated by traffic lights.



The first assumption underlying the traffic lights-model states that demand is fixed. The second assumption applies to the cycle time 'c', that is the duration of the sum of green time and red time. To simplify matters, the cycle time is held fixed. Hence, it follows that including inter-green time in the analysis is not relevant and will thus be ignored. The third assumption states that all users are identical and aim to minimize their expected travel cost.

In this paper, the variable 'r' represents the red time on route 2 and will be the main control variable. The corresponding green time on route 2 will thus be (c-r) and a reverse scenario holds for route 1.

Traffic conditions are usually under-saturated, whereby vehicle queues are only created during red phases, and are dissolved during green phases. Only during peak periods or at bottlenecks, users have to wait several cycles. In this paper the arrival rate is assumed to be static. This third assumption limits the model to under-saturated traffic conditions[1].

As there will be alternating red times to avoid collisions at the intersection, both routes will experience an expected traffic light waiting time cost $\left(T_1(c,r), T_2(c,r)\right)$. It is clear that the traffic light waiting cost functions are increasing in the red time and decreasing in the green time ($\frac{\partial T_1(c,r))}{\partial r} < 0$, $\frac{\partial T_2(c,r))}{\partial r} > 0$). Furthermore, whenever the red time approaches the cycle time, the waiting time tends to infinity ($\lim_{r \to 0} T_1(c,r) \to \infty, \lim_{r \to c} T_2(c,r) \to \infty$). In the reverse case, when the red time goes towards zero, the waiting time tends toward zero ($\lim_{r \to c} T_1(c,r) \to 0, \lim_{r \to 0} T_2(c,r) \to 0$). As we assume undersaturated traffic conditions, the expected traffic light waiting functions are discontinuous and jump to infinity when the duration of red time equals the total cycle time.

4. Results and discussion

Starting from the assumption that the traffic authority wants to minimize total travel cost, in each of the following sections the minimal total cost of the different policy decisions needs to be compared in order to determine the optimal policy.

4.1 Right of way

In this section, the intersection is regulated by a priority rule. First, route 1 is considered to have unlimited capacity, while route 2 is subject to linear congestion. Later, we

[1] Otherwise the queue would become infinitely long

consider the case where both routes have limited capacity. The reader is reminded that people using route 2 always have to give way to the users of route 1. Consequently, only the users of route 2 will incur a waiting time.

Proposition 1 *When the intersection of two routes connecting one OD-pair is regulated by a priority rule, the optimal policy is generally to block one of the two routes. The only exception is the case where the marginal congestion cost on the minor route is greater than half of the square of the marginal waiting cost ($a_2 > \frac{v^2}{2}$). In this scenario, the optimal policy is to leave both routes open.*

Only one route subject to congestion

Let a_1 represent the sensitivity to congestion on route 1; X_i equal the flow on route i; ω be the resource cost for a trip from A to C; φ_i stand for the minimal time cost from A to C of route i; and $\frac{v^2 X_1}{2}$ be the expected waiting time cost on route 2 at the intersection.

When two parallel routes connect one OD-pair, the government can lay down three possible policies: to block route 1, to block route 2, or to leave both routes open. The users in turn, can react in three different ways to any chosen policy: to only take route 1, to only use route 2, or to use both routes.

If the government decides to block route 1, then all drivers need to take route 2 and the total cost will equal $(\omega + \varphi_2)$ N. If, on the other hand, the government blocks route 2, then the user equilibrium will be X_1=N and the total cost will be (a_1N+ω +φ_1)N. If, however, the government decides to leave both routes open, the equilibrium reaction of the drivers will be X_1=N if a_1N+φ_1<φ_2+ $\frac{v^2(N-1)}{2}$, and X_2=N if φ_1>φ_2. If φ_1<φ_2< a_1N+φ_1-$\frac{v^2(N-1)}{2}$, the drivers will use both routes, and the Wardrop equilibrium implies (ω +φ_2+$\frac{v^2 X^e{}_2}{2}$)N as total cost. The user equilibrium in which both routes are used is, however, never optimal from the government's point of view. Therefore, it is always optimal to block one of the two routes.

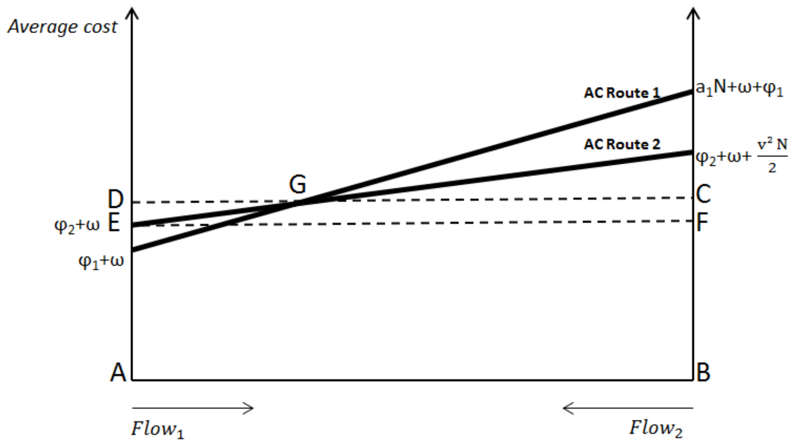
Which route to block depends on the relative cost of both routes: a rational authority minimizing the social cost closes route 1 if a_1N+φ_1>φ_2, and route 2 if a_1N+φ_1<φ_2.

It also follows from this that if the interior equilibrium exists, it is always optimal to block the route with limited capacity. This result can be explained intuitively. If both routes are used, drivers on the minor (uncongested) route incur a waiting cost, whereas if only the minor (uncongested) route were to be used, no waiting cost would be incurred and, compared to the interior solution, no other additional costs are incurred.

In the absence of a government intervention blocking one road, the driver will often make the sub-optimal choice. Indeed, whenever a_1N+φ_1-$\frac{v^2(N-1)}{2}$< φ_2< a_1N+φ_1, the user equilibrium is X_1=N, while X_2=N would be optimal. On the other hand, whenever φ_1<φ_2< a_1N+φ_1-$\frac{v^2(N-1)}{2}$ there will be an equilibrium in which both routes are used, while it would be optimal to have all drivers on route 2. Figure 3 illustrates this second situation. The Wardrop equilibrium is given by the intersection of the average cost-curves of route 2 and route 1 (point G)[2]. It is clear that the total cost for the interior solution equals ABCD, while the total costs would only equal ABFE if route 1 had been blocked.

[2] Note that for interior solutions every additional user on route 1 imposes an extra waiting cost for the drivers on route 2. This explains the upward sloping AC-curve of route 2 for increasing X_1

Figure 3: The solution in which all travellers use route 2 (E) is optimal. However, without intervention, the interior solution (G) will be the user equilibrium.



This can be seen as an illustration of the Braess paradox (Braess, 1968). In the Braess paradox, adding one additional link increases rather than decreases total travel cost. Here, we also add a link and it is the external waiting cost that users on the main road impose on the users of the minor road that can increase the total travel cost.

Both routes subject to congestion

The sub-optimality of an interior solution continues to hold for the case in which both routes are subject to congestion and $a_2 \leq \frac{v^2}{2}$ (with a_2 the sensitivity to congestion of route 2). However, when $a_2 \geq \frac{v^2}{2}$, the total costs are lowest when both routes are used in equilibrium.

Figure 4: The total travel costs resulting from a Stackelberg game

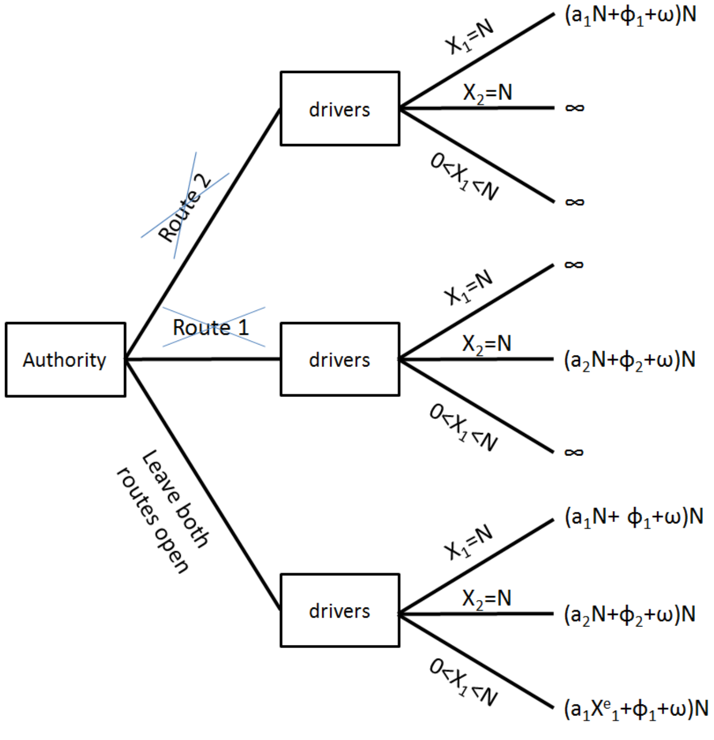


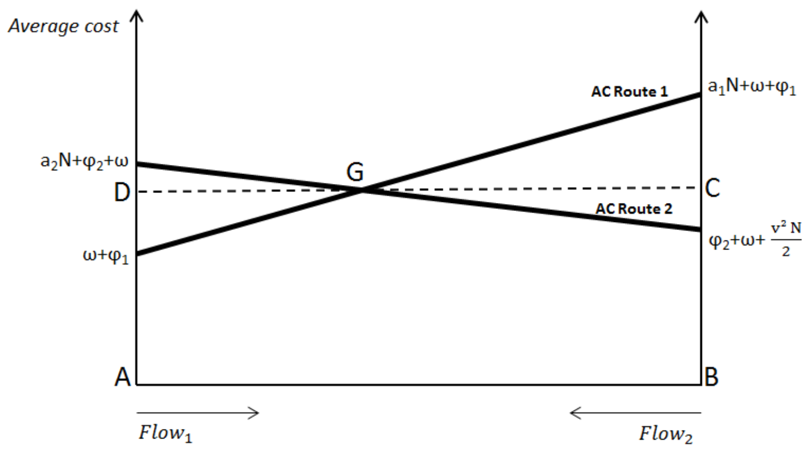
Figure 4 shows the different options for the government, and the possible reactions of the drivers. When the government decides to close route 1, the user equilibrium will be X_2=N and the total cost will be (a_2N+ω+φ_2)N. When only route 1 is accessible, X_1=N will be the only equilibrium and the total cost will amount to (a_1N+φ_1+ω)N. When both routes are accessible, the user equilibrium that will be in place depends on the relative value of the parameters: X_2=N if a_2N+φ_2<φ_1; X_1=N if a_1N+φ_1<φ_2+$\frac{v^2(N-1)}{2}$; and 0<$X^e{}_1$<N in all other cases.

Taking into account the reaction of the drivers, the government will block one of the two routes if $a_2 < \frac{v^2}{2}$ and leave both routes open if $a_2 \leq \frac{v^2}{2}$. The optimal policy thus depends on the ratio of the congestion coefficient (a_2) to the reaction time (v). If $a_2<\frac{v^2}{2}$, the situation is similar to the case where only one route is subject to congestion. If $a_2 \geq \frac{v^2}{2}$ and the government leaves both routes open, it can be shown that in the interior equilibrium the lowest cost will be attained.

In graphical terms, $a_2 < \frac{v^2}{2}$ boils down to an upward sloping (in X_1) average cost curve of route 2, while $a_2 \geq \frac{v^2}{2}$ boils down to a downward sloping AC_{route2}-curve.

The underlying logic is that if $a_2 \geq \frac{v^2}{2}$, the extra congestion cost of having all travelers on route 2 is more costly than the saving in waiting costs. If, however, $a_2 < \frac{v^2}{2}$ the opposite is true.

Figure 5: The interior solution (G) is optimal. No government intervention is needed to reach the optimal solution.



4.2 Traffic lights

In this subsection, the two-road intersection is regulated by traffic lights. Following the same approach as in the previous section, first only one route is considered to have limited capacity, and subsequently both routes are considered to have limited capacity. In this subsection, the government decides first on the signal settings and the drivers subsequently determine which route to take. The user equilibrium is again assumed to be a Nash equilibrium.

Only one route subject to congestion

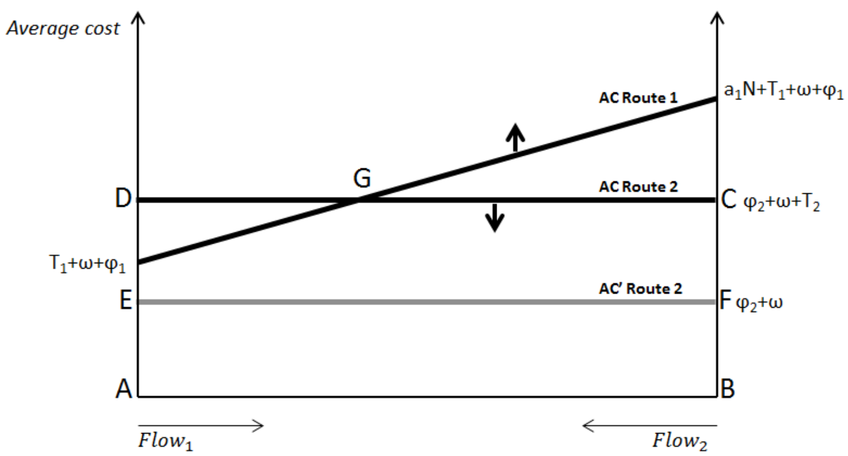
Proposition 2 *When the intersection of two routes connecting one OD-pair is regulated by traffic lights and only one of the two routes is congested, a signal setting whereby drivers choose to use both routes can never be an optimal policy.*

Table 1: The total travel cost for every (r,UE)-combination

Signal setting	X_1=N	X_2=N	0<X_1<N
r=c	$(a_1$N$+\varphi_1+\omega)$N	∞	∞
r=0	∞	$(\omega\ +\varphi_2)$ N	∞
0<r<c	$(a_1$N$+\varphi_1+\omega++T_1(c,r))$N	$(\omega\ +\varphi_2+T_2(c,r))$ N	$(\omega\ +\varphi_2+T_2(c,r))$ N

Table 1 shows that a rational government will never decide on an alternating signal setting when both routes are substitutes. Indeed, let $T_i(c,r)$ be the expected waiting time cost on route *i* at the intersection[3]. As $T_i(c,r)$ is positive when 0<r<c, the total travel cost for an alternating signal setting will always be higher than for r=c or r=0. Which of the two non-alternating signal settings will be optimal is dependent on the values of the parameters a_1, N, φ_1 and φ_2. Whenever a_1N+φ_1<φ_2, r=c is the optimal solution and whenever a_1N+$\varphi_1 \geq \varphi_2$, r=0 will be implemented.[4]

Figure 6: A corner solution constitutes the optimal solution; in this case: $X_2 = N$



The sub-optimality of an alternating signal (0<r<c) in case the intersection is regulated by traffic lights and only one route is subject to congestion, even though counter-intuitive, can be explained intuitively. When the user equilibrium is X_2=N or X_1=N (i.e. φ_2+$T_2(c,r)$< φ_1+$T_1(c,r)$ or a_1N+φ_1+$T_1(c,r)$<φ_2+$T_2(c,r)$ respectively), drivers have to wait at the traffic light, while there is no one crossing the intersection. The intuition behind the sub-optimality of an alternating signal setting when the user equilibrium is $X^e{}_1$<N is shown in Figure 5. If the duration of red light for route 2 is reduced, then the expected

[3] We will assume that the saturation flow rate g is very large compared to the arrival rate X_i, so that the traffic light waiting time due to departure delay in the green time is negligible.
[4] The optimality of only one route (mode) connecting one OD-pair remains valid in the case where both routes have unlimited capacity. The total minimal cost then equals $(\omega\ +\varphi_i)$N with *i* the route index that procures the lowest minimal time cost.

average waiting time on route 1 increases, indicated by an upward shift of the AC_{route1}-curve in Figure 5. At the same time, the expected average waiting time for route 2 decreases, corresponding to a downward shift of the AC_{route2}-curve curve. This forces the switching point G to the left, hence less people travel on route 1 with a simultaneous decrease in total costs.
From the graph it is clear that the lowest cost (area ABFE) will be obtained when it is always green for drivers on route 2 (grey line). It is noted that the AC_{route1}-curve would lie infinitely high in this situation.

This can be vivified by the following example: consider the situation in which the demand from A to B is relatively inelastic. Suppose A and B are connected by a road plagued by traffic congestion and a tram[5]. Users are indifferent between the two modes, only the user cost matters. The tram line intersects the road trajectory and this intersection is regulated by traffic lights. In this situation, even though counter-intuitive, the optimal policy would either be to close the road and only maintain the tram to connect point A and point B, or to remove the tram and only keep the road, depending on the relative costs of the two scenarios.

Both routes subject to congestion
Turning to the case in which congestion externalities are present on both routes, the analysis shows that in this case the optimal policy can be to leave both routes open.
Proposition 3 *When the intersection of two congested routes connecting one OD-pair is regulated by traffic lights, the optimal alternating signal setting is independent of the total flow and is given by* $\frac{a_2 c}{a_2+a_1}$.

Figure 7: Total costs when the combined assignment and control problem is modeled as a Stackelberg game.

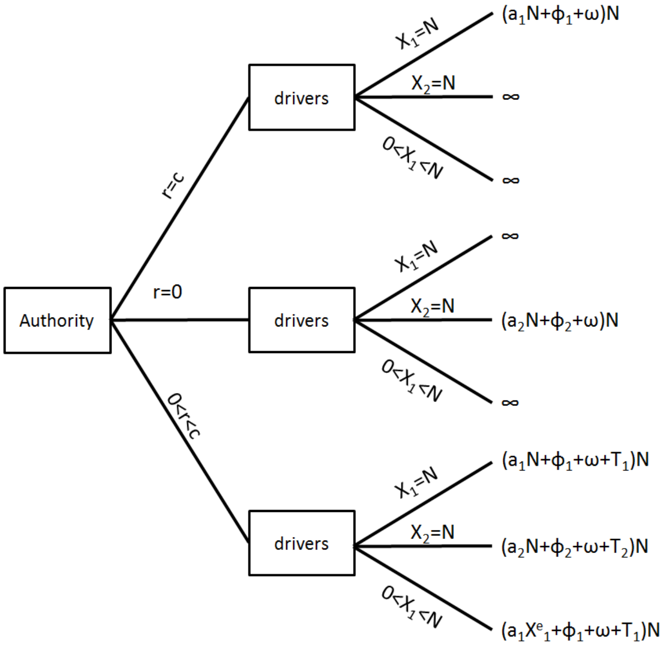


Figure 7 shows all the feasible signal settings and the reaction of the drivers to each of these signal settings. The third branch, representing the decision of the government to

[5] The tram is assumed to be relatively insensitive to congestion and a substitute for the car

implement an alternating signal setting (0<r<c), is a clubbing of all red times between zero and c.

It can be shown that if there exists an alternating signal setting for which the total cost is lower than for any non-alternating r (i.e., if $a_1 X^e{}_1 + T_1(c,r) < a_1$N and $a_2 X^e{}_2 + T_2(c,r) < a_2$N), then the user equilibrium for this r is the one in which both routes are used. Hence, the optimal alternating signal setting is the solution of the following optimization problem:

$$\min_{X_2, X_1, r} \big(a_1 X_1 + \omega + \varphi_1 + T_1(c,r)\big) X_1 + \big(a_2 X_2 + \omega + \varphi_2 + T_2(c,r)\big) X_2 \quad (1)$$

S.t.

$$X_1 + X_2 = N \quad (2)$$

$$a_1 X_1 + \omega + \varphi_1 + T_1(c,r) = a_2 X_2 + \omega + \varphi_2 + T_2(c,r) \quad (3)$$

$$0 \le r \le c \quad (4)$$

$$X_1 > 0 \quad (5)$$

$$X_2 > 0 \quad (6)$$

The solution will be determined by:

$$\frac{\partial T_2(c,r))}{\partial r} a_1 = -\frac{\partial T_1(c,r))}{\partial r} a_2 \quad (7)$$

Assuming under-saturated traffic conditions, i.e. queues at the intersection are only created during the red phases and dissolved during the green phases, the traffic light functions take the following form for 0 < r < c:

$$T_1(c,r) = \frac{(c-r)^2}{2c} \quad (8)$$

$$T_2(c,r) = \frac{r^2}{2c} \quad (9)$$

Inserting these expressions in equation (7), and solving for r, the optimal red time for route 2 equals:

$$r^* = \frac{a_2 c}{a_2 + a_1} \quad (10)$$

Even though the minimal time costs add to the total costs and we would therefore expect them to appear in the formula, they are not part of the optimal signal setting-formula. This can be explained by observing that the drivers themselves take the minimal time cost into account when choosing a route, while they omit the external congestion cost in their decision criterion. This omission is corrected by the optimal signal setting.

The optimal red time on route 2 increases in $\frac{a_2}{a_2+a_1}$. The more congestible is route 2 compared to route 1, the more users will take route 1 and thus a larger cost reduction is expected from an increase in green time on route 1.

The total costs that the optimal traffic light settings produce are given by the following equation:

$$\left(\frac{2a_1 a_2 (a_2+a_1)N + 2a_1(a_2+a_1)(\varphi_2-\varphi_1) + a_1 a_2 c}{2(a_2+a_1)^2} + \varphi_1 + \omega\right)\text{N} \quad (11)$$

If this cost is lower than (a_1N+φ_1+ω)N and (a_2N+ω+φ_2)N, then the optimal policy is to implement $r = \frac{a_2 c}{a_2+a_1}$. If on the other hand the parameters are such that (a_1N+φ_1+ω)N is the lowest cost, then the optimal policy would be to only give green to route 1. Finally, if (a_2N+ω+φ_2)N is the lowest cost a rational authority would implement r=0.

4.3 The choice between traffic lights and a priority rule

Proposition 5 *If only one route is subject to congestion, the superiority of a regulation by traffic lights over a priority rule becomes more likely the higher the flow on the main route*[6]*, the lower the response time of the drivers and the higher the cycle time. More specifically, for all* $r < \sqrt{v^2 c X_1}$ *it is optimal to opt for traffic lights.*

If for some reason both routes have to be used and if r can be chosen such that $r < \sqrt{v^2 c \frac{\varphi_2 - \varphi_1}{a_1 - \frac{v^2}{2}}}$ and 0<r<c, then traffic lights are the better choice.
This can be seen as follows: if both routes have to be used when traffic lights are present, the total cost amounts to $\left(\frac{r^2}{2c} + \omega + \varphi_2\right) N$. Comparing this with the total cost in a priority rule situation, $\left(\frac{v^2 X_1}{2} + \omega + \varphi_2\right) N$, it is clear that if $\frac{r^2}{2c} < \frac{v^2 X_1}{2}$ (this comes down to $r < \sqrt{v^2 c X_1}$), traffic lights reduce the total cost. The cost savings that accompany the transfer to traffic light regulation thus equal: $\left(\frac{v^2 X_1}{2} - \frac{r^2}{2c}\right) N$. If this value exceeds the annualized investment cost, traffic lights are optimal.
A further examination of the condition on r ($r < \sqrt{v^2 c \frac{\varphi_2 - \varphi_1}{a_1 - \frac{v^2}{2}}}$) shows that the choice for traffic lights is favoured by an increase in the usage of route 1 (higher X_1), and even more so by an increase in prudent driving behaviour (higher *v*). This is logical; suppose the usage of route 1 is very low, and the red time equals e.g. $\frac{c}{2}$, then cars on route 2 are often waiting while no car on route 1 crosses the intersection. Furthermore, if users on route 2 react quickly, then more users can pass the intersection, leading to a favourable regulation of the intersection by a priority rule.

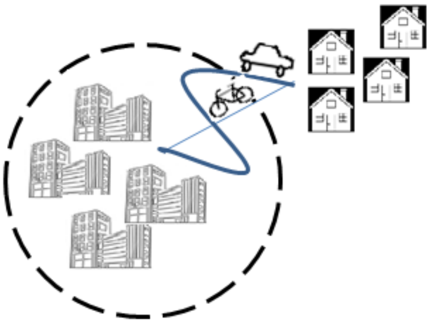
5. Two applications

In this section, some of the theoretical results obtained in the previous section are illustrated with an example.
The first example is an application of the following combination of results (see section 4.2): when two intersecting routes of which only one is congestible, connect one OD-pair and the interior equilibrium exists, then the optimal policy is to block the congested route. The second example extends the results in section 4.2 by including local traffic in the problem setting.

5.1 *A low-traffic city-center*

[6] which, on its turn, depends on the parameters φ_1, φ_2, v and a_1

Figure 8: Commuters can take the car or the bicycle.



Imagine a city where the inhabitants live on the edge of the city and work in the city center. There is a bike path as well as a congestible road connecting work and home and both are currently being used for commuting trips. In the city center, cyclists always have to give way to cars. Applying the results of section 4.2 to this situation, we can conclude that for this city the optimal policy would be to make the city center a car-free zone. This conclusion continues to stand if the crossing of the bicycle lane and the car road is regulated by traffic lights.

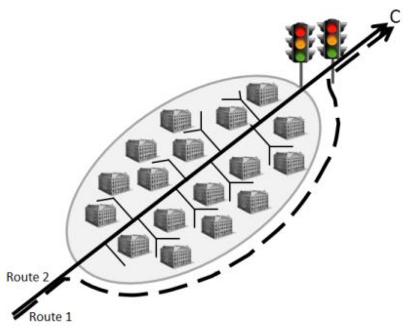
A city center in which no motorized traffic at all is allowed, is however unrealistic. After all, shops have to be provisioned and emergency vehicles have to be able to enter the city center. A simple solution, already adopted in many cities, is to allow only certain vehicles to enter the city center. This can be done using, for example, automatic rising bollards.

5.2 A bypass and a city road

A well-known situation in which one OD-pair is connected by two parallel roads is represented in Figure 9. Here, transit traffic has the choice between route 1 or route 2 to reach point C, while local traffic can only take route 2.

Let route 1 (the bypass) have a large capacity. Furthermore, we will assume that both local traffic and cut-through traffic contribute to the city road congestion. Suppose that the traffic lights are regulated by a federal authority whose objective it is to minimize the total cost of all drivers.

Figure 9: Transit traffic will either take the city road or the bypass depending on the signal settings.



Let X_b be the number of transit drivers taking the bypass per hour; X_v the number of transit drivers taking the city road per hour; a_v the congestion sensitivity of the city road;

φ_v the minimal time to go to point C using the city road; φ_b the minimal time to go to point C using the bypass; g the amount of cars that can cross the intersection per hour of green time; R the local traffic per hour; N the total transit traffic per hour; $T_i(c,r)$ the total waiting time at the traffic light on route *i*; and r the red time for the city road.

Table 2: Total travel cost for every (r,UE)-combination in a bypass situation.

Signal setting	X_b=N	X_v=N	0<X_b<N
r=c	∞	∞	∞
r=0	∞	$(a_v$(R+N)+$\varphi_v)$(N+R)	∞
0<r<c	$(a_v$R+$\varphi_v + T_v(c,r))$R +$(\varphi_b + T_b(c,r))$N	$(a_v$(R+N)+$\varphi_v + T_v(c,r))$(N+R)	$(\varphi_b + T_b(c,r))$ (N+R)

In Table 2 the total cost is shown for every combination of policy and user equilibrium. A glance at the table shows that a rational authority would never implement r=c. When the city road has always green (r=0), the only Nash equilibrium is X_v=N. The total cost in this case amounts to (φ_v+a_v(R+N))(R+N). Furthermore, it is remarked that the total cost for the combination (X_v=N, 0<r<c) is always larger than the total cost for the combination (X_v=N, r=0). Finally, it can be shown that the FOC of (φ_b+$T_b(c,r)$)(N+R) w.r.t. r is always negative[7], so the optimal solution entails that only one of the two routes is used.

The share of green time for the bypass is restricted by the fact that the local traffic has no choice but to use the city road. This maximal red time is an element of the interval for which X_b=N. Given that for this interval all transit traffic choses to use the bypass, it is welfare decreasing to give more than the minimal green time to the city-road. As a consequence, the maximal red time is the optimal r in this interval.

The previous observations narrow down the candidate solutions to either r=0 or r=$(g-R)\frac{c}{g}$. In order to determine the optimal signal setting, the government has to compare the cost of the (X_v=N)- and the (X_b=N)-solution.

It is clear that a local government, preferring a minimum of transit traffic in its city, would try to avoid the (X_v=N)-outcome. The local government can do this by increasing a_v or φ_v[8]. Increasing a_v or φ_v raises the cost of the (X_v=N, r=0)-combination relatively more[9], which decreases the likelihood of the federal government implementing r=0.

Despite the mathematical analysis required to obtain this result, many cities today already apply this strategy. Indeed, speed bumps and speed limits are put in place to increase φ_v and the capacity of roads is limited to increase a_v (De Borger and Proost (2013)).

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[7] So, the optimal r is the one for which the drivers are indifferent between using both routes or just the bypass.

[8] We only mention the parameters the local government can influence

[9] $\frac{dTC_{r=0}}{d\varphi_v}>\frac{dTC_{r=(g-R)\frac{c}{g}}}{d\varphi_v}$ and $\frac{dTC_{r=0}}{da_v}>\frac{dTC_{r=(g-R)\frac{c}{g}}}{da_v}$

6. Concluding remarks

In this paper, we studied the effects of a priority rule and traffic lights on an intersection of two routes connecting one OD-pair. We analytically minimized total travel cost, taking into account Wardrop's principles and the delay at the intersection. We have four major results: First, if the intersection is regulated by a priority rule, the optimal policy is generally to block one of the two routes; Second, if the intersection is regulated by traffic lights, and only one route is congestible the optimal policy is again to block one route. Third, if the intersection is regulated by traffic lights, the optimal alternating signal setting is always independent of the elasticity of demand. And finally, if only one route is subject to congestion, the superiority of a regulation by traffic lights over a priority rule becomes more likely the higher the flow on the main route, the lower the reaction time of the drivers and the higher the cycle time.
The results in this paper can be applied to solve one particular larger network problem. In this network problem the two routes are a chain of individual components similar to the one solved in this paper. If for every component it is optimal to use one and the same link[10], it can be concluded that it is optimal to maintain only one route. As this composition technique can only be applied to a certain kind of network problems, one of the future research opportunities is to extend the model to larger networks. Other future work includes the extension of the model toward saturated traffic conditions and an extension towards multiple government levels.

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[10] We can use the results from this paper to solve this problem on the component-level.